Energy Production in Stars*

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(Received September 7, 1938)

It is shown that the *most important source of energy in ordinary stars is the reactions of carbon and nitrogen with protons*. These reactions form a cycle in which the original nucleus is reproduced, *viz.* $C^{12}+H=N^{13}$, $N^{13}=C^{13}+\epsilon^+$, $C^{13}+H=N^{14}$, $N^{14}+H=O^{15}$, $O^{15}=N^{15}+\epsilon^+$, $N^{15}+H=C^{12}+He^4$. Thus carbon and nitrogen merely serve as catalysts for the combination of four protons (and two electrons) into an α-particle (§7).

The carbon-nitrogen reactions are unique in their cyclical character (§8). For all nuclei lighter than carbon, reaction with protons will lead to the emission of an α-particle so that the original nucleus is permanently destroyed. For all nuclei heavier than fluorine, only radiative capture of the protons occurs, also destroying the original nucleus. Oxygen and fluorine reactions mostly lead back to nitrogen. Besides, these heavier nuclei react much more slowly than C and N and are therefore unimportant for the energy production.

The agreement of the carbon-nitrogen reactions with observational data (§7, 9) is excellent. In order to give the correct energy evolution in the sun, the central temperature of the sun would have to be 18.5 million degrees while integration of the Eddington equations gives 19. For the brilliant star Y Cygni the corresponding figures are 30 and 32. This good agreement holds for all bright stars of the main sequence, but, of course, not for giants.

For fainter stars, with lower central temperatures, the reaction $H+H=D+\epsilon^+$ and the reactions following it, are believed to be mainly responsible for the energy production. (§10)

It is shown further (§5–6) that *no elements heavier than* He^4 *can be built up in ordinary stars*. This is due to the fact, mentioned above, that all elements up to boron are disintegrated by proton bombardment (α-emission!) rather than built up (by radiative capture). The instability of Be^8 reduces the formation of heavier elements still further. The production of neutrons in stars is likewise negligible. The heavier elements found in stars must therefore have existed already when the star was formed.

Finally, the suggested mechanism of energy production is used to draw conclusions about astrophysical problems, such as the mass-luminosity relation (§10), the stability against temperature changes (§11), and stellar evolution (§12).

§1. INTRODUCTION

THE progress of nuclear physics in the last few years makes it possible to decide rather definitely which processes can and which cannot occur in the interior of stars. Such decisions will be attempted in the present paper, the discussion being restricted primarily to main sequence stars. The results will be at variance with some current hypotheses.

The first main result is that, under present conditions, no elements heavier than helium can be built up to any appreciable extent. Therefore we must assume that the heavier elements were built up *before* the stars reached their present state of temperature and density. No attempt will be made at speculations about this previous state of stellar matter.
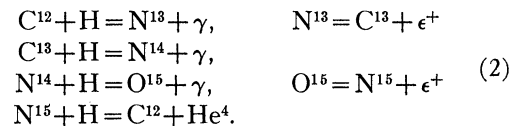
The energy production of stars is then due entirely to the combination of four protons and two electrons into an α-particle. This simplifies the discussion of stellar evolution inasmuch as

the amount of heavy matter, and therefore the opacity, does not change with time.

The combination of four protons and two electrons can occur essentially only in two ways. The first mechanism starts with the combination of two protons to form a deuteron with positron emission, *viz.*

$$H+H=D+\epsilon^+. \tag{1}$$
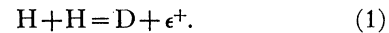
The deuteron is then transformed into He^4 by further capture of protons; these captures occur very rapidly compared with process (1). The second mechanism uses carbon and nitrogen as catalysts, according to the chain reaction

$$
\begin{aligned}
&C^{12}+H=N^{13}+\gamma, &&N^{13}=C^{13}+\epsilon^+\\
&C^{13}+H=N^{14}+\gamma,\\
&N^{14}+H=O^{15}+\gamma, &&O^{15}=N^{15}+\epsilon^+\\
&N^{15}+H=C^{12}+He^4.
\end{aligned}
\tag{2}
$$

The catalyst C^{12} is reproduced in all cases except about one in 10,000, therefore the abundance of carbon and nitrogen remains practically unchanged (in comparison with the change of the number of protons). The two reactions (1) and

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(2) are about equally probable at a temperature of $16 \cdot 10^6$ degrees which is close to the central temperature of the sun ($19 \cdot 10^6$ degrees[1]). At lower temperatures (1) will predominate, at higher temperatures, (2).

No reaction other than (1) or (2) will give an appreciable contribution to the energy production at temperatures around $20 \cdot 10^6$ degrees such as are found in the interior of ordinary stars. The lighter elements (Li, Be, B) would "burn" in a very short time and are not replaced as is carbon in the cycle (2), whereas the heavier elements (O, F, etc.) react too slowly. Helium, which is abundant, does not react with protons because the product, Li^5, does not exist; in fact, the energy evolution in stars can be used as a strong additional argument against the existence of He^5 and Li^5 (§3).

Reaction (2) is sufficient to explain the energy production in very luminous stars of the main sequence as Y Cygni (although there are difficulties because of the quick exhaustion of the energy supply in such stars which would occur on any theory, §9). Neither of the reactions (1) or (2) is capable of accounting for the energy production in giants; if nuclear reactions are at all responsible for the energy production in these stars it seems that the only ones which could give sufficient energy are

$$H^2 + H = He^3 \tag{3}$$

$$Li^{6, 7} + H = He^{3, 4} + He^4.$$
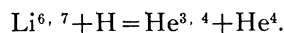
It seems, however, doubtful whether the energy production in giants is due to nuclear reactions at all.[2]

We shall first calculate the energy production by nuclear reactions (§2, 4). Then we shall prove the impossibility of building up heavier elements under existing conditions (§5–6). Next we shall discuss the reactions available for energy production (§5, 7) and the results will be compared with available material on stellar temperatures and densities (§8, 9). Finally, we shall discuss the astrophysical problems of the mass-luminosity relation (§10), the stability of stars against temperature changes (§11) and stellar evolution (§12).

[1] B. Strömgren, Ergebn. d. Exakt. Naturwiss. 16, 465 (1937).
[2] G. Gamow, private communication.

§2. FORMULA FOR ENERGY PRODUCTION

The probability of a nuclear reaction in a gas with a Maxwellian velocity distribution was first calculated by Atkinson and Houtermans.[3] Recently, an improved formula was derived by Gamow and Teller.[4] The total number of processes per gram per second is[4]

$$p = \frac{4}{3^{5/2}} \frac{\rho x_1 x_2}{m_1 m_2} \frac{\Gamma}{\hbar} a R^2 e^{4(2R/a)^{\frac{1}{2}}} \tau^2 e^{-\tau}. \tag{4}$$

Here ρ is the density of the gas, $x_1 x_2$ the concentrations (by weight) of the two reacting types of nuclei, $m_1 m_2$ their masses, $Z_1 e$ and $Z_2 e$ their charges, $m = m_1 m_2/(m_1+m_2)$ the reduced mass, R the combined radius,

$$a = \hbar^2/me^2 Z_1 Z_2 \tag{5}$$

the "Bohr radius" for the system, $\Gamma/\hbar$ the probability of the nuclear reaction, in sec.$^{-1}$, after penetration, and

$$\tau = 3\left(\frac{\pi^2 me^4 Z_1^2 Z_2^2}{2\hbar^2 kT}\right)^{\frac{1}{3}}. \tag{6}$$

If we measure ρ in g/cm^3, Γ in volts and T in units of 10^6 degrees, we have

$$p = 5.3 \cdot 10^{25} \rho x_1 x_2 \Gamma \varphi(Z_1, Z_2) \tau^2 e^{-\tau} \quad \text{g}^{-1}\text{sec.}^{-1}, \tag{7}$$

$$\tau = 42.7(Z_1 Z_2)^{\frac{2}{3}}(A/T)^{\frac{1}{3}}, \tag{8}$$

$$\varphi = \frac{1}{A_1 A_2 (Z_1 Z_2 A)^3}\left(\frac{8R}{a}\right)^2 e^{2(8R/a)^{\frac{1}{2}}}. \tag{9}$$

where $A_1 A_2$ are the atomic weights of the reacting nuclei ($A_i = m_i/m_H$), $A = m/m_H$, $m_H =$ mass of hydrogen. For the combined radius of nuclei 1 and 2 we put

$$R = 1.6 \cdot 10^{-13}(A_1 + A_2)^{\frac{1}{3}} \text{ cm}. \tag{10}$$

Then we obtain for φ the values given in Table I. The values of φ for isotopes of the same element differ only very slightly.

The values of Γ for reactions giving particles can be deduced from the *observed* cross sections

[3] R. d'E. Atkinson and F. G. Houtermans, Zeits. f. Physik 54, 656 (1929).
[4] G. Gamow and E. Teller, Phys. Rev. 53, 608 (1938), Eq. (3).

TABLE I. *Values of φ for various nuclear reactions.*

REACTION	R (10^{-13} CM)	φ	REACTION	R	φ
H^2+H	2.3	0.38	$Si^{30}+H$	5.0	29.3
H^3+H	2.5_5	0.48	$Cl^{37}+H$	5.4	75
He^4+H	2.7_5	0.81	H^2+H^2	2.5_5	0.67
Li^7+H	3.2	0.91	Be^7+H^2	3.3	1.18
Be^9+H	3.4_5	1.16	Be^7+He^3	3.4_5	7.9
$B^{11}+H$	3.6_5	1.52	He^4+H^2	2.9	0.57
$C^{12}+H$	3.7_5	2.00	He^4+He^3	3.0_5	1.09
$N^{14}+H$	3.9_5	2.78	He^4+He^4	3.2	1.29
$O^{16}+H$	4.1	3.80	Li^7+He^4	3.5_5	4.9
$F^{19}+H$	4.3_5	5.5	Be^7+He^4	3.5_5	13.2
$Ne^{22}+H$	4.5_5	7.7	Be^8+He^4	3.6_5	16.2
$Mg^{26}+H$	4.8	13.2	$C^{12}+He^4$	4.0	230

TABLE II. *Cross sections and widths for some nuclear reactions giving particles.*

REACTION	REF.	E KV	σ CM²	R CM	Γ
$H^2+H^2 = He^3+n^1$	5	100	$1.7\cdot10^{-26}$	$2.6\cdot10^{-13}$	$3\cdot10^5$
$Li^7+H^1 = 2He^4$	6	42	$1.7\cdot10^{-30}$	$3.2\cdot10^{-13}$	$4\cdot10^4$
$Li^6+H^1 = He^4+He^3$	yield $\sim$ same as Li^7+H in natural Li target				$5\cdot10^5$
$Li^6+H^2 = \begin{cases}2He^4\\Li^7+H\end{cases}$	7	212	$1.9\cdot10^{-26}$	$3.2\cdot10^{-13}$	$4\cdot10^6$
$Li^7+H^2 = 2He^4+n$	7	212	$5.5\cdot10^{-26}$	$3.3\cdot10^{-13}$	10^7
$Be^9+H^1 = \begin{cases}Li^6+He^4\\Be^8+H^2\end{cases}$	7a	212	$1.1\cdot10^{-25}$	$3.5\cdot10^{-13}$	$1.7\cdot10^7$
$Be^9+H^2 = \begin{cases}Li^7+He^4\\Be^8+H^3\\Be^{10}+H^1\end{cases}$	7a	212	$5\cdot10^{-28}$	$3.6\cdot10^{-13}$	$6\cdot10^5$
$B^{11}+H^1 = 3He^4$	7b	212	$6\cdot10^{-28}$	$3.7\cdot10^{-13}$	$2\cdot10^6$

of such reactions with the use of the formula (cf. reference 4, Eq. (2))

$$\sigma = \frac{\pi R^2}{2E}\frac{A_1+A_2}{A_2}\Gamma \exp\left[\left(\frac{32R}{a}\right)^{\frac{1}{2}} - \frac{2\pi e^2}{hv}Z_1Z_2\right] \quad (11)$$

where E is the absolute energy of the incident particle (particle 1). Table II gives the experimental results for some of the better investigated reactions. In each case, experiments with low energy particles were chosen in order to make the conditions as similar as possible to those in stars where the greatest number of nuclear reactions is due to particles of about 20 kilovolts energy. The cross sections were in each case calculated from the thick target yield with the help of the range-energy relation of Herb,

Bellamy, Parkinson and Hudson.[8] The widths obtained (last column of Table II) are mostly between $3\cdot10^5$ and $2\cdot10^7$ ev, with the exception of the reaction $Li^7+H = 2He^4$ which is known to be "improbable."[9]

The γ-ray widths Γ_γ can be obtained from observed resonance capture of protons. Table III gives the experimental results. Two of the older data were taken from Table XXXIX of reference 10; all the others are from more recent experiments on proton[11-14] and neutron[15] capture. Although the results of different investigators differ considerably (e.g., for $Li^7+H^1 = Be^8$, Γ is between 4 and 40 volts the latter value being more likely) they seem to lie generally between about $\frac{1}{2}$ and 40 volts. Ordinarily, the width is somewhat larger for the more energetic γ-rays, as is expected theoretically. A not too bad approximation to the experiments is obtained by using the theoretical formula for dipole radiation (reference 10, Eq. (711b)) with an oscillator strength of 1/50. This gives

$$\Gamma_\gamma \sim 0.1E_\gamma^2, \quad (12)$$

TABLE III. *γ-ray widths of nuclear levels.*

REACTION	REFERENCE	WIDTH (VOLTS)	γ-RAY ENERGY (MEV)
$Li^7+H^1 = Be^8+\gamma$	$\begin{cases}10\\11\end{cases}$	$\begin{matrix}4\\40\end{matrix}$	17
$B^{11}+H^1 = C^{12}+\gamma$	11	0.6	12, 16
$C^{12}+H^1 = N^{13}+\gamma$	12	0.6	2
$C^{13}+H^1 = N^{14}+\gamma$	13, 14	30	4, 8
$F^{19}+H^1 = Ne^{20}+\gamma$	10	0.6, 8, 18	6
$C^{12}+n^1 = C^{13}+\gamma$	15	<2.5	5
$O^{16}+n^1 = O^{17}+\gamma$	15	<2.5	4

[5] R. Ladenburg and M. H. Kanner, Phys. Rev. **52**, 911 (1937).

[6] H. D. Doolittle, Phys. Rev. **49**, 779 (1936).

[7] J. H. Williams, W. G. Shepherd and R. O. Haxby, Phys. Rev. **52**, 390 (1937).

[7a] J. H. Williams, R. O. Haxby and W. G. Shepherd, Phys. Rev. **52**, 1031 (1937).

[7b] J. H. Williams, W. H. Wells, J. T. Tate and E. J. Hill, Phys. Rev. **51**, 434 (1937).

[8] D. B. Parkinson, R. G. Herb, J. C. Bellamy and C. M. Hudson, Phys. Rev. **52**, 75 (1937).

[9] M. Goldhaber, Proc. Camb. Phil. Soc. **30**, 560 (1934).

[10] H. A. Bethe, Rev. Mod. Phys. **9**, 71 (1937).

[11] W. A. Fowler, E. R. Gaerttner and C. C. Lauritsen, Phys. Rev. **53**, 628 (1938).

[12] R. B. Roberts and N. P. Heydenburg, Phys. Rev. **53**, 374 (1938).

[13] P. I. Dee, S. C. Curran and V. Petržilka, Nature **141**, 642 (1938). The γ-rays from C^{13} give about equally as many counts as those from C^{12}. The efficiency of the counter for C^{13} γ-rays is about twice that for C^{12}, because the cross section for production of Compton and pair electrons is smaller by a factor 2/3 while the range of these electrons is about 3 times longer. With an abundance of C^{13} of about 1 percent, the γ-width for this nucleus becomes 50 times that of C^{12}. I am indebted to Dr. Rose for these calculations.

[14] M. E. Rose, Phys. Rev. **53**, 844 (1938).

[15] O. R. Frisch, H. v. Halban and J. Koch, Nature **140**, 895 (1937); Danish Acad. Sci. **15**, 10 (1937).

where E_γ is the γ-ray energy in mMU (milli-mass-units), and Γ_γ the γ-ray width in volts. For quadrupole radiation, theory gives about

$$\Gamma_\gamma \sim 5 \cdot 10^{-4} E_\gamma{}^4 \quad \text{(quadrupole).} \quad (12a)$$

Formulae (12), and (12a) will be used in the calculations where experimental data are not available; they may, in any individual case, be in error by a factor 10 or more but such a factor is not of great importance compared with other uncertainties.

It should be noted that quite generally radiative processes are rare compared with particle emission. According to the figures given in Tables II and III, the ratio of probabilities is 10^4–10^5 in favor of particle reactions.

In a number of cases, the reaction of a nucleus A with a heavy particle (proton, alpha-) must compete with natural β-radioactivity of A or with electron capture. In those cases where the lifetime of radioactive nuclei is not known experimentally, we use the Fermi theory. According to this theory, the decay constant for β-emission is[16]

$$\beta = 0.9 \cdot 10^{-4} f(W) |G|^2 \text{ sec.}^{-1}. \quad (13)$$

The matrix element G is about unity for strongly allowed transitions, and

$$f(W) = (W^2 - 1)^{\frac{1}{2}} \left(\frac{1}{30} W^4 - \frac{3}{20} W^2 - \frac{2}{15} \right)$$
$$+ \tfrac{1}{4} W \log \{ W + (W^2 - 1)^{\frac{1}{2}} \}, \quad (13a)$$

where W is the maximum energy of the β-particle, including its rest mass, in units of mc^2 (m = electron mass).

The probability of electron capture is

$$\beta_C = 0.9 \cdot 10^{-4} \pi^2 N (\hbar/mc)^3 W^2 |G|^2 \text{ sec.}^{-1}, \quad (14)$$

where W is the energy of the emitted neutrino in units of mc^2 and N the number of electrons per unit volume. If the hydrogen concentration is x_H, we have (reference 1, p. 482) $N = 6 \cdot 10^{23} \rho \cdot \frac{1}{2}(1 + x_H)$ (ρ the density), and

$$\beta_C = 1.5 \cdot 10^{-11} \rho (1 + x_H) W^2 |G|^2 \text{ sec.}^{-1}. \quad (14a)$$

§3. STABILITY OF UNKNOWN ISOTOPES

For the discussion of nuclear reactions it is essential to know whether or not certain isotopes exist (such as Li^4, Li^5, Be^6, Be^8, B^8, B^9, C^{10}, etc.). The criterion for the existence of a nucleus is its energetic stability against spontaneous disintegration into heavy particles (emission of a neutron, proton or alpha-particle). Whenever a light nucleus is energetically unstable against heavy particle emission, its life will be a very small fraction of a second (usually $\sim 10^{-20}$ sec.) even if the instability is slight (e.g., Be^8 will have a life of 10^{-13} sec. if it is by 50 kv heavier than two α-particles[17]).

For the question of the lifetime of radioactive nuclei, it is also necessary to know the mass difference between isobars. Similar information is required for estimating the γ-ray width in capture reactions (cf. Eqs. (12), (12a)).

H^3 and He^3

The most recent determination[18] of the reaction energy in the reaction $H^2 + H^2 = He^3 + n^1$ yielded 3.29 Mev as compared with 3.98 in $H^2 + H^2 = H^3 + H^1$. With a mass difference of 0.75 Mev between neutron and hydrogen atom,[19] this makes He^3 more stable than H^3 by 0.06 Mev. This would be in agreement with the experimental fact that no H^3 is present in natural hydrogen to more than 1 part in 10^{12}. Even if He^3 should turn out to be heavier than H^3, the difference cannot be greater than about 0.05 Mev = 0.1 mc^2 which would make the life of He^3 exceedingly long (~ 2000 years at the center of the sun, (Eq. (14a)), 3000 years in the complete atom, on earth).

H^4 and Li^4

As was first pointed out by Bothe and Gentner,[20] it is definitely possible that H^4 is stable. Li^4 is, of course, less stable because of the Coulomb repulsion between its three protons. If it is stable, Li^4 would be formed when He^3 captures a proton and would thus play an important role in stars (cf. §5). The only possible estimate of the stability seems to be a comparison

[16] C. L. Critchfield and H. A. Bethe, Phys. Rev. 54, 248, 862 (L) (1938).

[17] H. A. Bethe, Rev. Mod. Phys. 9, 167 (1937).
[18] T. W. Bonner, Phys. Rev. 52, 685 (1938).
[19] H. A. Bethe, Phys. Rev. 53, 313 (1938).
[20] W. Bothe and W. Gentner, Naturwiss. 24, 17 (1936).

of Li^4Li^5 to the analogous pair B^8B^9. Reasonable estimates[21] make B^9 just unstable (0.3–0.7 mMU), while B^8 comes out at the limit of stability (binding energy between -0.4 and $+0.4$ mMU), i.e., B^8 about 0.3 to 0.7 mMU more stable than B^9. Li^5 (see below) is unstable by 1.4–1.8 mMU; if one assumes the same difference, Li^4 is found to be unstable by ~1 mMU. However, this argument is very uncertain and the possibility of a stable Li^4 cannot be excluded at present. H^4 would, from a similar argument, turn out stable by 0.6 mMU.

He^5 and Li^5

The instability of He^5 is shown directly by the experiments of Williams, Shepherd and Haxby[22] on the reaction

$$Li^7 + H^2 = He^5 + He^4. \tag{15}$$
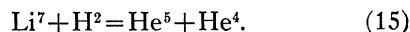
From a measurement of the range of the α-particles, the mass of He^5 is found (reference 23, Table 73, p. 373) to be 5.0137 whereas the combined mass of an α-particle plus a neutron would be only $4.003\ 86 + 1.008\ 93 = 5.012\ 8$. Thus He^5 is unstable by 0.9 mMU (milli-mass-units) which is far outside the experimental error (about 0.1–0.2 mMU). It might be argued that the α-particle group observed in reaction (15) might correspond to an excited state of He^5. However, this is extremely improbable because a nucleus of such a simple structure as He^5 (α-particle plus neutron) should not have any low-lying excited levels.[24] (This holds both in the α-particle and the Hartree model of nuclear structure.) Moreover, it would be difficult to explain why the α-particle group corresponding to the ground state of He^5 should not have been observed.

The conclusion that the mass of 5.0137 found by W. S. and H. really corresponds to the ground state of He^5 is supported by considerations of mass defects. In fact, the instability of He^5 was

first predicted by Atkinson[25] on the basis of such considerations. Considering analogous nuclei, consisting of α-particles plus one neutron, we find that the last neutron is bound with a binding energy of 5.3 mMU in C^{13} and with only 1.8 mMU in Be^9. A binding energy of -0.9 mMU in He^5 fits very well into this series while a positive binding energy would not.

If He^5 is unstable, this is a fortiori true of Li^5 since the binding energies of these two nuclei should differ by just the Coulomb repulsion between proton and alpha in Li^5. This repulsion will be about 0.6–1 mMU, so that Li^5 is unstable by 1.4–1.8 mMU.

Thus all the nuclear evidence[25a] points to the nonexistence of both He^5 and Li^5. Even if there were no such evidence, astrophysical data themselves would force us to this conclusion, because at a temperature of $2 \cdot 10^7$ degrees (central temperature of sun) the energy production from the combination of He^4 and H forming Li^5 would be of the order of 10^{10} ergs/g sec. (cf. §4), as against an observed energy production of 2 ergs/g sec. Only the nonexistence of Li^5 prevents this enormous production of energy.

Be^6

This nucleus is certainly unstable, as can be shown by comparing it with the known nucleus He^6 from which it differs by the interchange of protons and neutrons. The Coulomb energy which is the only difference between the binding energies of the two nuclei can be calculated rather accurately.[21] The instability against disintegration into $He^4 + 2H$ is between 1 and 2.6 mMU.

Be^7

This nucleus has been observed by Roberts, Heydenburg and Locher.[26] It decays with a

[21] H. A. Bethe, Phys. Rev. 54, 436, 955 (1938).

[22] J. H. Williams, W. G. Shepherd and R. O. Haxby, Phys. Rev. 51, 888 (1937).

[23] M. S. Livingston and H. A. Bethe, Rev. Mod. Phys. 9, 247 (1937).

[24] With the possible exception of a doublet structure of the ground state, similar to Li^7. However, the doublet separation should probably be much smaller than in Li^7 because of the loose binding of He^5, and presumably both components of the doublet are already contained in the rather broad α-particle group observed by Williams, Shepherd and Haxby.

[25] R. d'E. Atkinson, Phys. Rev. 48, 382 (1935).

[25a] Note added in proof:—Recently, F. Joliot and I. Zlotowski (J. de phys. et rad. 9, 403 (1938)) reported the formation of stable He^5 from the reaction $He^4 + H^2 = He^5 + H^1$. The evidence is based upon the emission of singly charged particles of long range when heavy paraffin is bombarded by α-particles. However, the number of such particles observed was exceedingly small (only 6 out of a total of 126 tracks). Furthermore, the mass given for He^5 by Joliot and Zlotowski (5.0106) is irreconcilable with the stability (against neutron emission) of the well-known nucleus He^6.

[26] R. B. Roberts, N. P. Heydenburg and G. L. Locher, Phys. Rev. 53, 1016 (1938).

TABLE IV. *Corrected and additional nuclear masses, and binding energies.*

NUCLEUS	MASS	BINDING ENERGY (mMU)	REFERENCE
n^1	1.008 93		19
He^3	3.016 99	5.87	18
H^4	4.025 4	0.6 ± 1	
He^4	4.003 86		29
Li^4	4.026 9	-1 ± 1	
He^5	5.013 7	-0.9 ± 0.2	23
Li^5	5.013 6	-1.6 ± 0.3	
Be^6	6.021 9	-1.8 ± 0.8	21
Be^7	7.019 28	5.7	26
Be^8	8.007 80	-0.08 ± 0.04	28
B^8	8.027 4	0.0 ± 0.4	21
B^9	9.016 4	-0.5 ± 0.2	21
C^{10}	10.020 2	3.8	21
N^{12}	12.022 5 -24 3	0.0 ± 0.9	21
N^{13}	13.010 08	2.03	19
O^{14}	14.013 1	5.1	21

half-life of 43 days (mean life 60 days) and probably only captures K electrons. Calculations of the Coulomb energy,[21] on the other hand, would make positron emission just possible (positron energy~ 0.1 mMU). As a compromise, we assume that the mass of Be^7 is just equal to Li^7 plus two electrons, i.e., 7.019 28.

Be^8

The instability of Be^8 against disintegration into two α-particles has been definitely established by Paneth and Glückauf[27] who have shown that the Be^8 formed in the photoelectric disintegration of Be^9 disintegrates into 2 He^4. Kirchner and Neuert[28] have confirmed this conclusion by investigating the products of the disintegration $B^{11}+H = Be^8+He^4$. They found that frequently two α-particles enter the detecting apparatus simultaneously, with a small angle (less than 50°) between their respective directions of flight; this is just what should be expected if the Be^8 formed breaks up into two α-particles on its way to the detector. From the average angle between the two alphas, the disintegration energy of Be^8 (difference Be^8-2He^4) was estimated as between 40 and 120 kev.[28a]

[27] F. A. Paneth and E. Glückauf, Nature **139**, 712 (1937).
[28] F. Kirchner and H. Neuert, Naturwiss. **25**, 48 (1937).
[28a] *Note added in proof:*—These conclusions are compatible with the new measurements of S. K. Allison, E. R. Graves, L. S. Skaggs and N. M. Smith, Jr. (Phys. Rev. **55**, 107 (1939)) on the reaction energy of $Be^9+H = Be^8+H^2$.
[29] K. T. Bainbridge, Phys. Rev. **53**, 922(A) (1938).

B^8

The existence is doubtful; calculation[21] by comparison with its isobar Li^8 gives a binding energy between -0.4 and $+0.4$ mMU. This nucleus is not very important for astrophysics.

B^9

B^9 is almost certainly unstable, as can be shown by calculating[21] the difference in binding energy (Coulomb energy) between it and its isobar Be^9. The theoretical instability is between 0.3 and 0.7 mMU, 0.3 being almost certainly a lower limit. However, in view of the smallness of this instability, we shall at least discuss what would happen if B^9 were stable (§6). It will turn out that this would make almost no difference at "ordinary" temperatures ($2\cdot 10^7$ degrees) and not much even at higher ones (10^8 degrees). For these calculations, we shall assume B^9 to be stable with 0.2 mMU which seems generous.

C^{10}

C^{10} is stable with 4 mMU against Be^8+2H.

N^{12}

N^{12} is doubtful, mainly because the binding energy of its isobar, B^{12}, is known only very inaccurately (between 2 and 3.3 mMU). Assuming 2 mMU, N^{12} would probably be instable, with 3.3 stable.

Table IV summarizes the binding energies of doubtful nuclei, and also gives some nuclear masses supplementary to and correcting those given in reference 23, p. 373.

§4. REACTION RATES AT $2\cdot 10^7$ DEGREES

We are now prepared to actually calculate the rate of nuclear reactions under the conditions prevailing in stars. We choose a temperature of twenty million degrees, close to the temperature at the center of the sun. In order to have a figure independent of density and chemical composition, we calculate (cf. 7)

$$P = (m_2/x_2)p/\rho x_1, \qquad (16)$$

$P\rho x_1$ gives the probability (per second) that a given nucleus of kind 2 undergoes a reaction with any nucleus of kind 1. If there are no other reactions destroying or producing nuclei of kind 2, $1/P\rho x_1$ will be the mean life of nuclei of kind 2 in the star.

TABLE V. *Probability of nuclear reactions at $2 \cdot 10^7$ degrees.***

REACTION	Q (MMU)	Γ (EV)		P (SEC.$^{-1}$)	LIFE, FOR $\rho x_1 = 30$
$H + H = H^2 + \epsilon^+$	1.53	Ref. 16	12.5	$8.5 \cdot 10^{-21}$	$1.2 \cdot 10^{11}$ yr.
$H^2 + H = He^3$	5.9	1 E	13.8	$1.3 \cdot 10^{-2}$	2 sec.
$H^3 + H = He^4$	21.3	10 E	14.3	$1.7 \cdot 10^{-1}$	0.2 sec.
$He^3 + H = Li^{4*}$	(0.5)	0.02 D	22.7	$3 \cdot 10^{-7}$	1 day
$He^4 + H = Li^{5*}$	(0.2)	0.005 D	23.2	$6 \cdot 10^{-8}$	6 days
$Li^6 + H = He^4 + He^3$	4.1	$5 \cdot 10^5$ X	31.1	$7 \cdot 10^{-3}$	5 sec.
$Li^7 + H = 2\ He^4$	18.6	$4 \cdot 10^4 X$	31.3	$6 \cdot 10^{-4}$	1 min.
$Be^7 + H = B^8?$	(0.5)	0.02 D	38.1	$6 \cdot 10^{-13}$	2000 yr.
$Be^9 + H = Li^6 + He^4$	2.4	10^6 X	38.1	$4 \cdot 10^{-5}$	15 min.
$B^9 + H = C^{10*}$	3.5	2 D	44.6	$2 \cdot 10^{-13}$	5000 yr.
$B^{10} + H = C^{11}$	9.2	10 D	44.6	10^{-12}	1000 yr.
$B^{11} + H = 3\ He^4$	9.4	10^6 E	44.6	$1.2 \cdot 10^{-7}$	3 days
$C^{11} + H = N^{12}$	(0.4)	0.02 D	50.6	10^{-17}	10^8 yr.
$C^{12} + H = N^{13}$	2.0	0.6 X	50.6	$4 \cdot 10^{-16}$	$2.5 \cdot 10^6$ yr.
$C^{13} + H = N^{14}$	8.2	30 X	50.6	$2 \cdot 10^{-14}$	$5 \cdot 10^4$ yr.
$N^{14} + H = O^{15}$	7.8	5 D	56.3	$2 \cdot 10^{-17}$	$5 \cdot 10^7$ yr.
$N^{15} + H = C^{12} + He^4$	5.2	10^5 E	56.3	$5 \cdot 10^{-13}$	2000 yr.
$O^{16} + H = F^{17}$	0.5	0.02 D	61.6	$8 \cdot 10^{-22}$	10^{12} yr.
$F^{19} + H = O^{16} + He^4$	8.8	10^5 E	66.9	$4 \cdot 10^{-17}$	$3 \cdot 10^7$ yr.
$Ne^{22} + H = Na^{23}$	10.7	10 D	71.7	$5 \cdot 10^{-23}$	$2 \cdot 10^{13}$ yr.
$Mg^{26} + H = Al^{27}$	8.0	10 D	81.3	10^{-26}	10^{17} yr.
$Si^{30} + H = P^{31}$	7.0	10 D	90.4	$4 \cdot 10^{-30}$	$3 \cdot 10^{20}$ yr.
$Cl^{37} + H = A^{38}$	12.0	10 D	103.1	$5 \cdot 10^{-35}$	$2 \cdot 10^{25}$ yr.
$H^2 + H^2 = He^3 + n$	3.5	$3 \cdot 10^5$ X	15.7	10^3	
$Be^7 + H^2 = B^{9*}$	18.5	10 D'	45.9	$2 \cdot 10^{-13}$	
$Be^7 + H^3 = B^9 + n^*$	11.9	10^6 E	50.7	$2 \cdot 10^{-10}$	
$Be^7 + He^3 = C^{10}$	16.2	1 D'	80.5	$3 \cdot 10^{-28}$	
$H^2 + He^4 = Li^6$	1.7	$4 \cdot 10^{-3}$ Q	27.5	$3 \cdot 10^{-10}$	
$He^3 + He^4 = Be^7$	1.6	0.02 D'	47.3	$3 \cdot 10^{-17}$	$3 \cdot 10^7$ yr.
$He^4 + He^4 = Be^{8*}$	(0.05)	$5 \cdot 10^{-9}$ Q	50.0	10^{-24}	
$Li^7 + He^4 = B^{11}$	9.1	1 D'	71.0	$2.5 \cdot 10^{-24}$	
$Be^7 + He^4 = C^{11}$	8.0	1 D'	86	$3 \cdot 10^{-30}$	$3 \cdot 10^{20}$ yr.
$C^{12} + He^4 = O^{16}$	7.8	1 Q'	119	$7 \cdot 10^{-43}$	

** The letters in the column giving the level width mean: X =experimental value; D =calculated for dipole radiation, from Eq. (12); D' = dipole radiation with small specific charge, 1/4 to 1/20 of Eq. (12); Q =quadrupole radiation, Eq. (12a); and E =estimate.
* These reactions are not believed to occur since their product or one of the reactants is unstable. They are listed merely for the sake of discussion.

Table V gives the results of the calculations, based on Eqs. (7) to (9). In the first column, the nuclear reactions are listed. All reactions which seemed of importance in the interior of stars were considered; in addition, some reactions with heavier elements (O^{16} to Cl^{37}) were included in order to show the manner in which the reaction rate decreases. Moreover, seven reactions were listed in spite of the fact that their products or reactants are believed to be (§3) unstable (starred) or doubtful (question mark); these reactions are included in order to discuss the consequences if they did occur.

The second column gives the energy evolution Q in the reaction, calculated from the masses (reference 23, Table LXXIII, and this paper, Table IV). In the third column, the width Γ determining the reaction rate (cf. §2) is tabulated. Wherever possible, this was taken from experiments (Tables II and III) or from the "empirical formulae" (12), (12a) for the radia-

tion width. For the radiative combination of two nuclei of equal specific charge ($H^2 + He^4$, $He^4 + He^4$, $C^{12} + He^4$) quadrupole radiation was assumed, otherwise dipole radiation.[30] For almost equal specific charge (e.g., $Be^7 + He^4$), the dipole formula with an appropriate reduction was used. In some instances, the width was estimated by analogy (e.g., $N^{15} + H = C^{12} + He^4$) or from approximate theoretical calculations ($H^2 + H = He^3$).[31] The way in which Γ was obtained was indicated by a letter in each instance.

The fourth column contains τ, as calculated from (8), the fifth P from (16). The wide variation of P is evident, also the smallness of P for α-particle as compared with proton reactions.

[30] If the combined initial nuclei and the final nucleus have the same parity (as may be the case, e.g., for $O^{16} + H$ $= F^{17}$), it is still possible to have a dipole transition if only the incident particle has orbital momentum one. This does not materially affect its penetrability if $R > a$ (cf. (4), (5)) which is true in every case where the parities are expected to be the same.
[31] L. I. Schiff, Phys. Rev. **52**, 242 (1937).

E.g., the reaction between He^4 and a nucleus as light as Be is as improbable as between a proton and Si. This arises, of course, from the greater charge and mass of the α-particle both of which factors reduce its penetrability. The reaction $He^4 + He^4 = Be^8$ has an exceedingly small probability because of the small frequency and the quadrupole character of the emitted γ-rays. Thus this reaction would not be important even if Be^8 were stable. On the other hand, the reaction $He^4 + H = Li^5$ would be extremely probable if Li^5 existed. The helium in the sun would be "burnt up" completely in about six days, even if rather unfavorable assumptions are made about the probability of the reaction. Similarly, if the energy evolution per process is 0.2 mMU $= 3 \cdot 10^{-7}$ ergs, the energy produced per gram of the star would be

$$(6 \cdot 10^{23}/4)\rho x_H x_{He} \cdot 3 \cdot 10^{-7} \cdot 6 \cdot 10^{-8}.$$

With $\rho = 80$, $x_H = 0.35$ and $x_{He} = 0.1$, this would give about 10^{10} ergs/g sec. as against 2 ergs/g sec. observed. This is a very strong additional argument against the existence of Li^5.

In the last column of Table V, the mean life is calculated for the various nuclei reacting with protons, by assuming a density $\rho = 80$ and hydrogen content $x_1 = 35$ percent, which correspond to the values at the center of the sun.[1] It is seen that, with the exception of H, the lifetimes of all nuclei up to boron are quite short, ranging from a fraction of a second for H^3 to 1000 years for B^{10}. (The life of B^{10} may actually be slightly shorter because of the reaction $B^{10} + H = Be^7 + He^4$. See §6.) Of the two lives longer than 1000 years listed, one refers to B^9 which probably does not exist (§3), the other to Be^7 which decays by positron emission with a half-life of 43 days.[26] We must conclude that *all the nuclei between H and C, notably* H^2, H^3, Li^6, Li^7, Be^9, B^{10}, B^{11}, *can exist in the interior of stars only to the extent to which they are continuously re-formed by nuclear reactions*. This conclusion does not apply to He^4 because Li^5 does not exist. To He^3 it probably applies whether Li^4 exists or not, because He^3 will also be destroyed by combination with He^4 into Be^7, although with a considerably longer period ($3 \cdot 10^7$ years instead of the 1 day for the reaction giving Li^4).

The actual lifetime of carbon and nitrogen is much longer than it would appear from the table because these nuclei are reproduced by the nuclear reactions themselves (§7). This makes their actual lifetime of the order of 10^{12} (or even 10^{20}, cf. §7) years, i.e., long compared with the age of the universe ($\sim 2 \cdot 10^9$ years). Protons, and all nuclei heavier than nitrogen, also have lives long compared with astronomical times.

§5. The Reactions Following Proton Combination

In the last section, it has been shown that all elements lighter than carbon, with the exception of H^1 and He^4, have an exceedingly short life in the interior of stars. Such elements can therefore only be present to the extent to which they are continuously produced in nuclear reactions from elements of longer life. This is in accord with the small abundance of all these elements both in stars and on earth.

Of the two more stable nuclei, He^4 is too inert to play an important rôle. It combines neither with a proton nor with another α-particle since the product would in both cases be an unstable nucleus. The only way in which He^4 can react at all, is by triple collisions. These will be discussed in the next section and will be shown to be very rare, as is to be expected.

As the only primary reaction between elements lighter than carbon, there remains therefore the reaction between two protons,

$$H^1 + H^1 = H^2 + \epsilon^+. \tag{1}$$
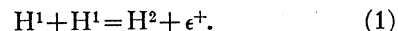
According to Critchfield and Bethe,[16] this process gives an energy evolution of 2.2 ergs/g sec. under "standard stellar conditions" ($2 \cdot 10^7$ degrees, $\rho = 80$, hydrogen content 35 percent). The reaction rate under these conditions is (cf. Table V) $2.5 \cdot 10^{-19}$ sec.$^{-1}$, corresponding to a mean life of $1.2 \cdot 10^{11}$ years for the hydrogen in the sun. This lifetime is about 70 times the age of the universe as obtained from the red shift of nebulae.

According to the foregoing, any building up of elements out of hydrogen will have to start with reaction (1). The deuteron will capture another proton,

$$H^2 + H^1 = He^3. \tag{17}$$

This reaction follows almost instantaneously upon (1), with a delay of only 2 sec. (Table V). There is, therefore, always statistical equilibrium between protons and deuterons, the concentrations (by number of atoms) being in the ratio of the respective lifetimes. This makes the concentration of deuterons (by weight) equal to

$$x(H^2) = x(H^1) \cdot \frac{2 \cdot 8.5 \cdot 10^{-21}}{1.3 \cdot 10^{-2}}$$

$$= 1.3 \cdot 10^{-18} x(H^1) \quad (18)$$

(cf. Table V). The relative probability of the reaction
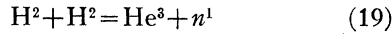
$$H^2 + H^2 = He^3 + n^1 \quad (19)$$

as compared with (17), is then

$$p_n = \frac{1}{4} \cdot \frac{P(H^2 + H^2 = He^3 + n^1)}{P(H^2 + H = He^3)} \cdot \frac{x(H^2)}{x(H^1)}$$

$$= \frac{10^3 \cdot 1.3 \cdot 10^{-18}}{4 \cdot 1.3 \cdot 10^{-2}} = 2 \cdot 10^{-14}. \quad (19a)$$

(One factor $\frac{1}{2}$ comes from the fact that in (19) two nuclei of the same kind interact; another is the atomic weight of H^2.) Thus one neutron is produced for about $5 \cdot 10^{13}$ proton combinations.
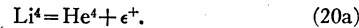
The further development of the He^3 produced according to (17) depends on the question of the stability of Li^4 and of the relative stability of H^3 and He^3.

Assumption A: Li⁴ stable

In this case, the He^3 will capture another proton, viz.:

$$He^3 + H^1 = Li^4. \quad (20)$$

With the assumptions made in Table V, the mean life of He^3 would be 1 day. The Li^4 would then emit a positron:

$$Li^4 = He^4 + \epsilon^+. \quad (20a)$$

With an assumed stability of Li^4 of 0.5 mMU compared with $He^3 + H^1$, the maximum energy of the positrons in (20a) would be 20.8 mMU = 19.4 Mev (including rest mass) which would be by far the highest β-ray energy known. The lifetime of Li^4 may accordingly be expected to be a small fraction of a second (half-life = 1/500 sec. for an allowed transition in the Fermi theory).

The most important consequence of the stability of Li^4 would be that only a fraction of the mass difference between four protons and an α-particle would appear as usable energy. For in the β-emission (20a) the larger part of the energy is, on the average, given to the neutrino which will in general leave the star without giving up any of its energy

(see below). According to the Konopinski-Uhlenbeck theory, which is in good agreement with the observed energy distribution in β-spectra, the neutrino receives on the average 5/8 of the total available energy if the latter is large. In our case, this would be 13.0 mMU. Adding 0.2 mMU for the neutrino emitted in process (1), we find that altogether 13.2 mMU energy is lost to neutrinos, of a total of 28.7 mMU developed in the formation of an α-particle out of four protons and two electrons. Thus the observable energy evolution is only 15.5 mMU, i.e., 54 percent of the total. Therefore, if Li^4 is stable, process (1) would give only 1.2 ergs/g sec. instead of 2.2 (under "standard" conditions).[16]

The neutrinos emitted will have some chance of producing neutrons in the outer layers of the star. It seems reasonable to assume that a neutrino has no other interaction with matter than that implied in the β-theory. Then a free neutrino (ν) will cause only "reverse β-processes"[32] of which the simplest and most probable is

$$H + \nu = n^1 + \epsilon^+. \quad (21)$$

This process is endoergic with 1.9 mMU and is therefore caused only by fast neutrinos such as those from Li^4. The cross section is according to the Fermi theory

$$\sigma = \pi^2 (\hbar/mc)^3 \cdot 0.9 \cdot 10^{-4} c^{-1} W (W^2 - 1)^{\frac{1}{2}}$$
$$= 1.7 \cdot 10^{-45} W (W^2 - 1)^{\frac{1}{2}} \text{ cm}^2, \quad (22)$$

where W is the energy of the emitted positron, including rest mass, in units of mc^2. In reaction (21), this is the neutrino energy minus 1.35 mMU. For the Li^4 neutrinos, the average cross section comes out to be

$$\sigma_{Av} = 2.5 \cdot 10^{-43} \text{ cm}^2 \quad (22a)$$

per proton, and the probability of process (21) for a neutrino starting from the center of the star

$$p = 6 \cdot 10^{23} \cdot \sigma_{Av} x_H \int_0^R \rho dr = 1.5 \cdot 10^{-19} x_H \int_0^R \rho dr, \quad (22b)$$

where $\rho(r)$ is the density (in g/cm³) at the distance r from the center of the star. For the sun, $p = 1.6 \cdot 10^{-7}$. This means that $1.6 \cdot 10^{-7}$ of the neutrinos emitted will cause reaction (21) before leaving the sun, and that the number of neutrons formed is $1.6 \cdot 10^{-7}$ times the number of proton combinations (1).

A further consequence of (20, 20a) would be that ordinarily no nuclei heavier than 4 mass units are formed at all, even as intermediate products. Such nuclei would only be produced in the rare cases when H^2 or He^3 capture an α-particle rather than a proton, according to the reactions

$$H^2 + He^4 = Li^6 \quad (23)$$

and

$$He^3 + He^4 = Be^7. \quad (24)$$

Under the favorable assumption that the concentration of He^4 is the same as of H^1 (by weight), the fraction of H^2 forming Li^6 is (cf. Table V)

$$p(Li^6) = 3 \cdot 10^{-10} / 1.3 \cdot 10^{-2} = 2 \cdot 10^{-8} \quad (23a)$$

the fraction of He^3 giving Be^7 is

$$p(Be^7) = 3 \cdot 10^{-17} / 3 \cdot 10^{-7} = 10^{-10}. \quad (24a)$$

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[32] H. A. Bethe and R. Peierls, Nature **133**, 689 (1934).

Most of the Li^6 will give rise to the well-known reaction

$$Li^6 + H = He^3 + He^4 \tag{23b}$$

and most of the Be^7 will go over into Li^7 which in turn reacts with a proton to give two α-particles. Only occasionally, Li^6, Be^7 or Li^7 will capture an α-particle and thus form heavier nuclei. It can be shown (cf. assumption B) that Li^7 is the most efficient nucleus in this respect. Therefore, the amount of heavier elements formed is determined by Be^7, the mother substance of Li^7, and is thus 10^{-10} times the amount formed with assumption B.

Assumption B: Li^4 unstable, He^3 more stable than H^3

This assumption seems to be the most likely according to available evidence. The *only* reaction which the He^3 can undergo, is then (24), i.e., each proton combination leads to the formation of a Be^7 nucleus. The most probable mode of decay of this nucleus is by electron capture, leading to Li^7. The lifetime of Be^7 (half-life) is 43 days[26] in the complete atom, and 10 months at the center of the sun (cf. 14, 14a). This makes the mean life = 14 months and the reaction rate $2.8 \cdot 10^{-8}$ sec.$^{-1}$. The capture of a proton by Be^7 would, even if the product B^8 is stable, be 2000 times slower (Table V). Each electron capture by Be^7 is accompanied by the emission of a neutrino of energy $\sim 2mc^2 = 1.1$ mMU (when Li^7 is left in its excited state, which happens rather rarely, the neutrino receives only 0.6 mMU). The total energy lost to neutrinos (including process 1) will therefore be very small in this case (~ 1.3 mMU per α-particle formed, i.e., $4\frac{1}{2}$ percent of the total energy evolution) and practically the full mass energy will be transformed into heat radiation. The Li^7 formed by electron capture of Be^7 will cause the well-known reaction

$$Li^7 + H = 2\ He^4 \tag{25}$$

and have (Table V) a mean life of only 1 minute at $2 \cdot 10^7$ degrees.

The reaction chain described leads, as in the case of assumption A, to the building up of one α-particle out of four protons and two electrons, for each process (1). No nuclei heavier than He^4 are formed permanently. Such nuclei can be produced only by branch reactions alternative to the main chain described. These will be discussed in the following.

a. Reactions with protons.—When Li^7 reacts with slow protons, the result is not always two α-particles, but, in one case out of about[33] 5000, radiative capture, giving Be^8. However, Be^8 will disintegrate again into two α-particles (§3), and during its life of about 10^{-13} sec., the probability of its reacting with another particle (e.g., capture of another proton) is exceedingly small ($\sim 10^{-24}$). Similarly, Be^7 will, in one out of about 2000 cases (see above) capture a proton and form B^8 if that nucleus exists. However, B^8

[33] It was assumed that radiative capture takes place only through the resonance level at 440 kv proton energy. The proton width of this level was taken as 11 kv, the radiation width as 40 ev.

will again go over into Be^8, by positron emission, and two alphas will again be the final result.

At this place, obviously, stability of Be^8 would increase the yield of heavy nuclei. Then one stable Be^8 would be formed for 5000 proton combinations; and, if B^9 is also assumed to be stable, every Be^8 goes over into B^9. Since about one out of $3 \cdot 10^8$ B^9 gives a C^{12} (§6), the number of heavy nuclei (C^{12}) formed would be $\sim 10^{-12}$ per α-particle. This would be the highest yield obtainable. However, Be^8 is known to be unstable (§3).

b. Reactions with α-particles.—The only abundant light nucleus other than the proton is He^4. The only reaction possible between an α-particle and Li^7 or Be^7 is radiative capture, viz.

$$Li^7 + He^4 = B^{11}, \tag{26}$$

$$Be^7 + He^4 = C^{11}. \tag{26a}$$

The probability of formation of B^{11} and C^{11} is (Table V)

$$p(B^{11}) = \frac{P(Li^7 + He^4)}{P(Li^7 + H)} = \frac{2.5 \cdot 10^{-24}}{6 \cdot 10^{-4}} = 4 \cdot 10^{-21}, \tag{26b}$$

$$p(C^{11}) = \frac{P(Be^7 + He^4)}{P(Be^7 + \epsilon = Li^7)} = \frac{14\ \text{months}}{3 \cdot 10^{20}\ \text{yr.}} = 4 \cdot 10^{-21}. \tag{26c}$$

Thus the formation of B^{11} is about as probable as that of C^{11}; the effect of the lower potential barrier of Li^7 for α-particles is compensated by its shorter life. The C^{11} will, of course, also give B^{11} by positron emission.

The B^{11} will react with protons in two ways, viz.

$$B^{11} + H^1 = 3\ He^4, \tag{27}$$

$$B^{11} + H^1 = C^{12}. \tag{27a}$$

The branching ratio is about $10^4 : 1$ in favor of (27) (calculated from experimental data). Thus there will be one C^{12} nucleus formed for about 10^{24} α-particles. The building up of heavier nuclei, even in this most favorable case, is therefore exceedingly improbable.

c. Reactions with He^3.—Since He^3 has a rather long life $3 \cdot 10^7$ years, Table V) and penetrates more easily through the potential barrier than the heavier He^4, it may be considered as an alternative possibility. However, the probability of formation of C^{10} from $Be^7 + He^3$ is only 100 times greater than that of C^{11} from $Be^7 + He^4$ (Table V) if the concentrations of He^3 and He^4 are equal. Actually, that of He^3 is only about $3 \cdot 10^{-4}$ (life of He^3 divided by life of protons) so that this process is 1/30 as probable as (26a). For $Li^7 + He^3$, the situation is even less favorable.

d. Reactions with H^2.—Deuteron capture by Be^7 would lead to B^9 whose existence is very doubtful. The probability per second would be (cf. Table V and Eq. (18))

$$2 \cdot 10^{-13} \rho x(H^2) = 2 \cdot 10^{-13} \cdot 30 \cdot 1.3 \cdot 10^{-18} = 10^{-29}, \tag{28}$$

which is only 1/10 of the probability of (26a) (Table V). Moreover, most of the B^9 formed reverts to He^4 (§6) so that the contribution of this process is negligible.

e. Reactions with He^4 in statu nascendi.—The process (25) produces continuously fast α-particles which need not penetrate through potential barriers. These α-particles have a range of 8 cm each in standard air, corresponding to 16 cm = 0.02 g/cm^2 for both. In stars, with their large hydrogen content, a somewhat smaller figure must be used

since hydrogen has a greater stopping power per gram; we take 0.01 g/cm². The cross section for fast particles is about

$$\sigma = \pi R^2 \frac{\Gamma}{\hbar^2/MR^2}. \tag{29}$$

With $\Gamma = 1$ volt (Table V) and $R = 3.6 \cdot 10^{-13}$ cm (Eq. (10)), this gives $\sigma = 1.3 \cdot 10^{-31}$ cm². The number of Be⁷ atoms per gram is

$$6 \cdot 10^{23} x(\mathrm{Be}^7) = 6 \cdot 10^{23} \cdot \frac{14 \text{ months}}{1.2 \cdot 10^{11} \text{ yr}} x(\mathrm{H}) = 2 \cdot 10^{12} \tag{29a}$$

with $x(\mathrm{H}) = 0.35$. This gives for the number of processes (26a) per proton combination:

$$0.01 \cdot 2 \cdot 10^{12} \cdot 1.3 \cdot 10^{-31} = 2.5 \cdot 10^{-21}, \tag{29b}$$

which is about the same as the formation of C¹¹ or B¹¹ by capture of *slow* alphas (cf. 26b, c).

Returning to the main reaction chain in the case of our assumption B, we note that the formation of Be⁷ (Eq. (24)) is a very slow reaction, requiring $3 \cdot 10^7$ years at "standard" conditions ($2 \cdot 10^7$ degrees). At lower temperatures, the reaction will be still slower and, finally, it will take longer than the past life of the universe ($\sim 2 \cdot 10^9$ years). In this case, the amount of He³ present will be much smaller than its equilibrium value (provided there was no He³ "in the beginning") and the energy production due to reactions (24), (25) will be reduced accordingly. Ultimately, at very low temperatures ($< 12 \cdot 10^6$ degrees), the reaction H+H will lead only to He³, and will therefore give an energy production of only 7.2 mMU, i.e., only one-quarter of the high temperature value, 27.4 mMU.

Assumption C: H³ more stable than He³

In this case, He³ will be able to capture an electron,

$$\mathrm{He}^3 + \epsilon^- = \mathrm{H}^3. \tag{30}$$

Under the assumption that a difference in mass of 0.1 electron mass exists between He³ and H³, the probability of (30) is, according to (14a),

$$p(\mathrm{H}^3) = 1.5 \cdot 10^{-11} \text{ sec.}^{-1} \tag{30a}$$

for a density $\rho = 80$ and 35 percent hydrogen content. This corresponds to a mean life of ~ 2000 years. The electron capture is therefore about 10^4 times more probable than the formation of Be⁷ according to (24). This ratio will be reversed at temperatures $> 4 \cdot 10^7$ since (30) is independent of T and the probability of (24) increases as T^{15}.

H³ will capture a proton and form He⁴,

$$\mathrm{H}^3 + \mathrm{H} = \mathrm{He}^4$$
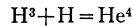
with a mean life of about 0.2 seconds. This way of formation of He⁴ from reaction (1) is probably the most direct of all. As in B, practically no energy is lost to neutrinos.

The formation of heavier elements goes as in B, but now there is only one Be⁷ formed for 10^4 proton combination processes. This reduces the probability of formation of C¹² by another factor 10^4, to one C¹² in 10^{28} alphas.
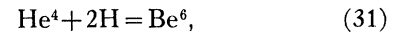
The H³ itself does not contribute appreciably to the building up of elements. It is true that the reaction Be⁷+H³ $= \mathrm{B}^9 + n^1$ is about 100 times as probable as Be⁷+H² $= \mathrm{B}^9$

(considering the shorter life of H³), and therefore 10 times as probable as (26a). However, most of the B⁹ reverts to He⁴ (cf. §6) so that (26) and (26a) remain the most efficient processes for building up C¹².

Summarizing, we find that the formation of nuclei heavier than He⁴ can occur only in negligible amounts. One C¹² in 10^{24} α-particles and one neutron in 10^{14} α-particles are the yields when Li⁴ is unstable, one C¹² in 10^{34} alphas and one neutron in 10^7 alphas when Li⁴ is stable. The reason for the small probability of formation of C¹² is twofold: First, any nonradioactive nucleus between He and C, i.e., Li⁶, ⁷, Be⁹, B¹⁰, ¹¹, reacting with protons will give α-particle emission rather than radiative capture so that a disintegration takes place rather than a building up. This will no longer be the case for heavier nuclei so that for these a building up is actually possible. Second, the instability of Be⁸ causes a gap in the list of stable nuclei which is the harder to bridge because Be⁸ is very easily formed in nuclear reactions (small mass excess). On the other hand, the instability of He⁵ and Li⁵ is of no influence because Be⁷ and Li⁷ are stages in the ordinary chain of nuclear reactions.

§6. TRIPLE COLLISIONS OF ALPHA-PARTICLES

In the preceding section, we have shown that collisions with protons alone lead practically always to the formation of α-particles. In order that heavier nuclei be formed, use must therefore certainly be made of the α-particles themselves. However, collisions of an α-particle with one other particle, proton or alpha, do not lead to stable nuclei. Therefore we must assume triple collisions, of which three types are conceivable:

$$\mathrm{He}^4 + 2\mathrm{H} = \mathrm{Be}^6, \tag{31}$$

$$2\mathrm{He}^4 + \mathrm{H} = \mathrm{B}^9, \tag{32}$$

$$3\mathrm{He}^4 = \mathrm{C}^{12}. \tag{33}$$

The first of these reactions leads to a nucleus which is certainly unstable (Be⁶). Even if it were stable, it would not offer any advantages over Be⁷ which is formed as a consequence of the proton combination (1). The second reaction leads to B⁹ which is probably also unstable. However, since this is not absolutely certain, we

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[34] G. Breit and E. Wigner, Phys. Rev. **49**, 519 (1936).

shall discuss this process in the following. The last process leads directly to C^{12}, but since it involves a rather large potential barrier for the last α-particle, it is very improbable at $2 \cdot 10^7$ degrees (see below).

The formation of B^9

The probability of this process is enhanced by the well-known resonance level of Be^8, which corresponds to a kinetic energy of about $E = 50$ kev of two α-particles. The formation of B^9 occurs in two stages,

$$2 He^4 = Be^8, \quad Be^8 + H = B^9 \tag{32a}$$
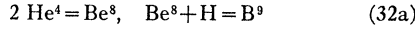
with a time interval of about 10^{-13} sec. (life of Be^8). The process can be treated with the usual formalism for resonance disintegrations, the compound nucleus being Be^8. This nucleus can "disintegrate" in two ways, (a) into two α-particles, (b) with proton capture. We denote the respective widths of the Be^8 level by Γ_α and Γ_H; the latter is given by the ordinary theory of thermonuclear processes, i.e.,

$$\Gamma_H = h p m_2 / x_2, \tag{34}$$

where p is given by (4), and the subscripts 1 and 2 denote H^1 and Be^8, respectively.

The cross section of the resonance disintegration becomes then

$$\sigma = \pi \lambda^2 \frac{\Gamma_\alpha \Gamma_H}{(E - E_r)^2 + \frac{1}{4}(\Gamma_\alpha + \Gamma_H)^2}. \tag{35}$$

E_r is the resonance energy. Γ_α is much larger than Γ_H (corresponding to about 10^{13} and 10^{-11} sec.$^{-1}$, respectively) but very small compared with E_r (about 10^{-2} against $5 \cdot 10^4$ volts). The resonance is thus very sharp, and, integrating over the energy, we obtain for the total number of processes per cm^3 per sec. simply

$$\rho p = B(E_r) v_r \pi \lambda_r^2 2 \pi \Gamma_H. \tag{36}$$

Here $B(E)dE$ is the number of pairs of α-particles with relative kinetic energy between E and $E + dE$ per cm^3, viz.

$$B(E) = \frac{1}{2} \left(\frac{\rho x_\alpha}{m_\alpha} \right)^2 \frac{2}{\pi^{\frac{1}{2}}} \frac{E^{\frac{1}{2}}}{(kT)^{\frac{3}{2}}} e^{-E/kT} \tag{36a}$$

(x_α = concentration of He^4 by weight). Combining (36), (36a) and (34), (4), we find

$$p(B^9) = \frac{16 \pi^{3/2}}{3^{5/2}} \frac{\rho^2 x_\alpha^2 x_H}{m_\alpha^{7/2} m_H} \frac{h^2 \Gamma_{\text{rad}}}{(kT)^{3/2}} a R^2 \tau^2$$
$$\times \exp \{4(2R/a)^{\frac{1}{2}} - \tau - E_r/kT\}, \tag{37}$$

where Γ_{rad} is the radiation width for the process $Be^8 + H = B^9$. Numerically, (37) gives for the decay constant of hydrogen the value

$$p m_H / x_H = 1.00 \cdot 10^{-4} (\rho x_\alpha)^2 \Gamma \varphi T^{-\frac{1}{2}} e^{-11.7 E_r / T} \tau^2 e^{-\tau}, \tag{37a}$$

where T is measured in millions of degrees, the resonance energy E_r of Be^8 in kilo-electron-volts and Γ in ev. The quantities Γ, φ and τ refer to the process $Be^8 + H = B^9$. If

there were no resonance level of Be^8, (37) would be replaced by

$$p(B^9) = \frac{8}{243} \frac{\rho^2 x_\alpha^2 x_H}{m_\alpha^2 m_H} \frac{\Gamma_{\text{rad}}}{\hbar} a R^2 a' R'^2 \tau^2 \tau'^2$$
$$\times \exp \{(32R/a)^{\frac{1}{2}} + (32R'/a')^{\frac{1}{2}} - \tau - \tau'\}, \tag{38}$$

where the primed quantities refer to the reaction $2 He^4 = Be^8$, the unprimed ones to $Be^8 + H = B^9$. Numerically, (38) gives

$$p m_H / x_H = 2.0 \cdot 10^{-11} (\rho x_\alpha)^2 \Gamma \varphi \tau^2 e^{-\tau} \varphi' \tau'^2 e^{-\tau'}. \tag{38a}$$

Assuming $T = 20$, $\rho = 80$, $x_\alpha = 0.25$, $\Gamma = 0.02$ electron-volts, we obtain for the probability of formation of B^9 per proton per second:

$$p m_H / x_H = 2 \cdot 10^{-25} \text{ for resonance, } E_r = 25 \text{ kev}$$
$$10^{-31} \text{ for resonance, } E_r = 50 \text{ kev}$$
$$4 \cdot 10^{-38} \text{ for resonance, } E_r = 75 \text{ kev}$$
$$2 \cdot 10^{-44} \text{ for resonance, } E_r = 100 \text{ kev}$$
$$5 \cdot 10^{-42} \text{ for nonresonance.}$$

The value 25 kev for the resonance level must probably be excluded on the basis of the experiments of Kirchner and Neuert.[28] But even for this low value of the resonance energy, the probability of formation of B^9 is only 10^{-6} times that of the proton combination $H + H = H^2 + \epsilon^+$ (Table V, $\rho x_1 = 30$). With $E_r = 50$ kev which seems a likely value, the ratio becomes $4 \cdot 10^{-13}$. On the other hand, the building up of B^9 (if this nucleus exists) would still be the most efficient process for obtaining heavier elements (see below).

Reactions of B^9

It can easily be seen[21] that B^9 cannot be positron-active but can only capture electrons if it exists at all. If B^9 is stable by 0.3 mMU, the energy evolution in electron capture would be just one electron mass. The decay constant of B^9 (for β-capture) is then, according to (14a), $1.5 \cdot 10^{-9}$ sec.$^{-1}$ ($\rho = 80$, $x_H = 0.35$) corresponding to a lifetime of about 20 years. On the other hand, the lifetime with respect to proton capture (Table V) is 5000 years. Therefore, ordinarily B^9 will go over into Be^9. This nucleus, in turn, will in general undergo one of the two well-known reactions:

$$Be^9 + H = Be^8 + H^2, \tag{39}$$
$$Be^9 + H = Li^6 + He^4. \tag{39a}$$
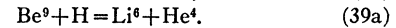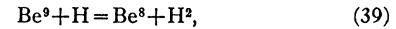
Only in one out of about 10^5 cases, B^{10} will be formed by radiative proton capture. Therefore the more efficient way for building up heavier elements will be the direct proton capture by B^9, leading to C^{10}, which occurs in one out of about 300 cases.

The C^{10} produced will go over into B^{10} by positron emission. B^{10} may react in either of the following ways:

$$B^{10} + H = C^{11}, \tag{40}$$
$$B^{10} + H = Be^7 + He^4. \tag{40a}$$
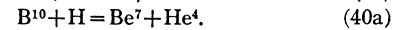
The reaction energy of (40a) is (cf. Table VII, §8) 1.2 mMU; the penetrability of the outgoing alphas about 1/40 (same table), therefore the probability of the particle reaction (40a) will be about 100 times that of the capture reaction (40).

The C^{11} from (40) will emit another positron. The resulting B^{11} reacts with protons as follows:

$$B^{11}+H=C^{12}, \tag{41}$$
$$B^{11}+H=3\ He^4. \tag{41a}$$
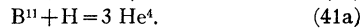
Both reactions are well-known experimentally. Reaction (41) has a resonance at 160 kev. From the width of this resonance and the experimental yields, the probability of (41) with low energy protons is about 1 in 10,000 (i.e., the same as for nonresonance). Altogether, about one B^9 in $3\cdot10^8$ will transform into C^{12}.

With a resonance energy of Be^8 of 50 kev, and $2\cdot10^7$ degrees, there will thus be about one C^{12} formed for 10^{21} α-particles if B^9 is stable. This is better than any other process but still negligibly small.

At higher temperatures, the formation of B^9 will become more probable and will, for $T>10^8$, exceed the probability of the proton combination. At these temperatures (actually already for $T>3\cdot10^7$) the B^9 will rather capture a proton (giving C^{10}). Even then, there remain the unfavorable branching ratios in reactions (40), (40a) and (41), (41a),[35] so that there will still be only one C^{12} formed in 10^6 alphas. Thus even with B^9 stable and granting the excessively high temperature, the amount of heavy nuclei formed is extremely small.

Direct formation of C^{12}

C^{12} may be formed directly in a collision between 3 α-particles. The calculation of the probability is exactly the same as for the formation of B^9. The nonresonance process gives about the same probability as a resonance of Be^8 at 50 kev. With $\rho=80$, $x_\alpha=\frac{1}{4}$, $\Gamma=0.1$ electron-volt, $T=2\cdot10^7$ degrees, the probability is 10^{-56} per α-particle, i.e., about 10^{-37} of the proton combination reaction (1). This gives an even smaller yield of C^{12} than the chains described in this and the preceding section. The process is strongly temperature-dependent, but it requires temperatures of $\sim10^9$ degrees to make it as probable as the proton combination (1).
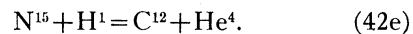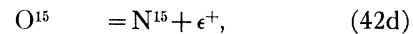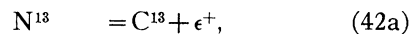
The considerations of the last two sections show that there is no way in which nuclei heavier than helium can be produced permanently in the interior of stars under present conditions. We can therefore drop the discussion of the building up of elements entirely and can confine ourselves to the energy production which is, in fact, the only observable process in stars.

§7. The Carbon-Nitrogen Group

In contrast to lighter nuclei, C^{12} is not permanently destroyed when it reacts with protons;

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[35] The reaction $C^{11}+H=N^{12}$ becomes more probable than $C^{11}=B^{11}+\epsilon^+$ only at $T>3\cdot10^8$ degrees. The branching ratio in (40), (40a) may perhaps be slightly more favorable because the effect of the potential barrier in (40a) may be stronger.

instead the following chain of reactions occurs

$$C^{12}+H^1=N^{13}, \tag{42}$$
$$N^{13}\qquad=C^{13}+\epsilon^+, \tag{42a}$$
$$C^{13}+H^1=N^{14}, \tag{42b}$$
$$N^{14}+H^1=O^{15}, \tag{42c}$$
$$O^{15}\qquad=N^{15}+\epsilon^+, \tag{42d}$$
$$N^{15}+H^1=C^{12}+He^4. \tag{42e}$$

Thus the C^{12} nucleus is reproduced. The reason is that the alternative reactions producing α-particles, *viz.*

$$C^{12}+H^1=B^9+He^4, \tag{43}$$
$$C^{13}+H^1=B^{10}+He^4, \tag{43a}$$
$$N^{14}+H^1=C^{11}+He^4, \tag{43b}$$

are all strongly forbidden energetically (Table VII, §8). This in turn is due to the much greater stability of the nuclei in the carbon-nitrogen group as compared with the beryllium-boron group, and is in contrast to the reactions of Li, Be and B with protons which all lead to emission of α-particles.

The cyclical nature of the chain (42) means that practically no carbon will be consumed. Only in about 1 out of 10^4 cases, N^{15} will capture a proton rather than react according to (42e). In this case, O^{16} is formed:

$$N^{15}+H^1=O^{16}. \tag{44}$$

However, even then the C^{12} is not permanently destroyed, because except in about one out of $5\cdot10^7$ cases, O^{16} will again return to C^{12} (cf. §8). Thus there is less than one C^{12} permanently consumed for 10^{12} protons. Since the concentration of carbon and nitrogen, according to the evidence from stellar spectra, is certainly greater than 10^{-12} this concentration does not change noticeably during the evolution of a star. *Carbon and nitrogen are true catalysts; what really takes place is the combination of four protons and two electrons into an α-particle.*

A given C^{12} nucleus will, at the center of the sun, capture a proton once in $2.5\cdot10^6$ years (Table V), a given N^{14} once in $5\cdot10^7$ years. These times are short compared with the age of the sun. Therefore the cycle (42) will have re-

TABLE VI. *Central temperatures necessary for giving observed energy production in sun, with various nuclear reactions.*

REACTION	T (MILLION DEGREES)
$H^2 + H = He^3$	0.36
$He^4 + H = Li^5$	2.1
$Li^7 + H = 2He^4$	2.2
$Be^9 + H = Li^6 + He^4$	3.3
$B^{10} + H = C^{11}$	9.2
$B^{11} + H = 3He^4$	5.5
$C^{12} + H = N^{13}$	15.5
$N^{14} + H = O^{15}$	18.3
$O^{16} + H = F^{17}$	32
$Ne^{22} + H = Na^{23}$	37

peated itself many times in the history of the sun, so that statistical equilibrium has been established between all the nuclei occurring in the cycle, *viz.* $C^{12}C^{13}N^{13}N^{14}N^{15}O^{15}$. In statistical equilibrium, the concentration of each nucleus is proportional to its lifetime. Therefore N^{14} should have the greatest concentration, C^{12} less, and $C^{13}N^{15}$ still less. (The concentration of the radioactive nuclei N^{13} and O^{15} is, of course, very small, about 10^{-12} of N^{14}). A comparison of the observed abundances of C and N is not very conclusive, because of the very different chemical properties. However, a comparison of the isotopes of each element should be significant.

In this respect, the result for the carbon isotopes is quite satisfactory. C^{13} captures slow protons about 70 times as easily as C^{12} (experimental value!), therefore C^{12} should be 70 times as abundant. The actual abundance ratio is 94 : 1. The same fact can be expressed in a more "experimental" way: In equilibrium, the number of reactions (42) per second should be the same as of (42b). Therefore, if a natural sample of terrestrial carbon (which is presumed to reproduce the solar equilibrium) is bombarded with protons, equally many captures should occur due to each carbon isotope. This is what is actually found experimentally;[13] the equality of the γ-ray intensities from C^{12} and C^{13} is, therefore, not accidental.[36]

The greater abundance of C^{12} is thus due to the smaller probability of proton capture which in turn appears to be due to the smaller $h\nu$ of

the capture γ-ray. Thus the great energetic stability of C^{12} actually makes this nucleus abundant. However, it is not because of a Boltzmann factor as has been believed in the past, but rather because of the small energy evolution of the proton capture reaction.

In nitrogen, the situation is different. Here N^{14} is energetically less stable (has higher mass excess) than N^{15} but is more abundant in spite of it (abundance ratio $\sim 500 : 1$). This must be due to the fact that N^{15} can give a $p - \alpha$ reaction while N^{14} can only capture a proton; particle reactions are always much more probable than radiative capture. Thus the greater abundance of N^{14} is due not to its own small mass excess but to the large mass excess of C^{11} which would be the product of the $p - \alpha$ reaction (43b).

Quantitative data on the nitrogen reactions (42c), (42e) are not available, the figures in Table V are merely estimates. If our theory about the abundance of the nitrogen isotopes is correct, the ratio of the reaction rates should be 500 : 1, i.e., either $N^{14} + H = O^{15}$ must be more probable[36a] or $N^{15} + H = C^{12} + He^4$ less probable than assumed in Table V. Experimental investigations would be desirable.

Turning now to the energy evolution, we notice that the cycle (42) contains two radioactive processes (N^{13} and O^{15}) giving positrons of 1.3_5 and 1.8_5 mMU maximum energy, respectively. If we assume again that $\frac{5}{8}$ of the energy is, on the average, given to neutrinos, this makes 2.0 mMU neutrino energy per cycle, which is 7 percent of the total energy evolution (28.7 mMU). There are therefore $4.0 \cdot 10^{-5}$ ergs available from each cycle. (It may be mentioned that the neutrinos emitted have too low energy to cause the transformation of protons into neutrons according to (21).)

The duration of one cycle (42) is equal to the sum of the lifetimes of all nuclei concerned, i.e., practically to the lifetime of N^{14}. Thus each N^{14} nucleus will produce $4.0 \cdot 10^{-5}$ erg every $5 \cdot 10^7$ years, or $3 \cdot 10^{-20}$ erg per second. Under the

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[36] It would be tempting to ascribe similar significance to the equality of intensity of the two α-groups from natural Li bombarded by protons ($Li^6 + H = He^4 + He^3$, $Li^7 + H = 2He^4$). However, the lithium isotopes do not seem to be genetically related as are those of carbon.

[36a] *Note added in proof:*—In this case, the life of N^{14} in the sun might actually be shorter, and its abundance smaller than that of C^{12}. Professor Russell pointed out to me that this would be in better agreement with the evidence from stellar spectra. Another consequence would be that a smaller abundance of N^{14} would be needed to explain the observed energy production.

TABLE VII. p–α reactions.

INITIAL NUCLEUS	PRODUCT NUCLEUS	ENERGY EVOLUTION Q (mMU)	POTENTIAL BARRIER B (mMU)	PENETRABILITY P
Li^6	He^3	4.14		1
Li^7	He^4	18.59		1
Be^9	Li^6	2.45	2.6	1
B^{10}	Be^7	1.2	3.4_5	0.027
B^{11}	$2He^4$	9.4		1
C^{12}	B^9	−8.1		0
C^{13}	B^{10}	−4.4		0
N^{14}	C^{11}	−3.5		0
N^{15}	C^{12}	5.2	4.6	1
O^{16}	N^{13}	−5.8		0
O^{17}	N^{14}	1.3	5.1	$1.7 \cdot 10^{-4}$
O^{18}	N^{15}	3.1	5.0	0.2
F^{19}	O^{16}	8.8		1
Ne^{20}	F^{17}	−4.5		0
Ne^{21}	F^{18}	−1.6		0
Ne^{22}	F^{19}	−1.6		0
Na^{23}	Ne^{20}	1.6	6.7	$6 \cdot 10^{-6}$
Mg^{24}	Na^{21}	<−3.0		0
Mg^{25}	Na^{22}	−2.1		0
Mg^{26}	Na^{23}	−2.0		0
Al^{27}	Mg^{24}	1.8	7.6	$1.1 \cdot 10^{-6}$
Si^{28}	Al^{25}	<−2.9		0
Si^{29}	Al^{26}	−2.0		0
Si^{30}	Al^{27}	−2.4		0
P^{31}	Si^{28}	2.0	8.5	$2.5 \cdot 10^{-7}$
S^{32}	P^{29}	<0		0
S^{33}	P^{30}	−2		0
S^{34}	P^{31}	−2.0		0
Cl^{35}	S^{32}	2.3	9.3	$5 \cdot 10^{-8}$
Cl^{37}	S^{34}	4.2	9.1	$2.5 \cdot 10^{-3}$

assumption of a N^{14} concentration of 10 percent by weight, this gives an energy evolution of

$$\frac{6 \cdot 10^{23} \cdot 0.1}{14} \cdot 3 \cdot 10^{-20} \approx 100 \text{ ergs/g sec.} \quad (45)$$

at "standard stellar conditions," i.e., $T = 2 \cdot 10^7$, $\rho = 80$, hydrogen concentration 35 percent.

This result is just about what is necessary to explain the observed luminosity of the sun. Since the nitrogen reaction depends strongly on the temperature (as T^{18}) and the temperature, as well as the density, decrease rapidly from the center of the sun outwards, the average energy production will be only a fraction, perhaps 1/10 to 1/20, of the production at the center.[36b] This means that the average production is 5 to 10 ergs/g sec., in excellent agreement with the observed luminosity of 2 ergs/g sec.

[36b] *Added in proof:*—According to calculations of R. Marshak, the correct figure is about 1/30.

Thus we see that the reaction between nitrogen and protons which we have recognized as the logical reaction for energy production from the point of view of nuclear physics, also agrees perfectly with the observed energy production in the sun. This result can be viewed from another angle: We may ignore, for the moment, all our nuclear considerations and ask simply which nucleus will give us the right energy evolution in the sun? Or conversely: Given an energy evolution of 20 ergs/g sec. at the center of the sun, which nuclear reaction will give us the right central temperature ($\sim 19 \cdot 10^6$ degrees)?

This calculation has been carried out in Table VI. It has been assumed that the density is 80, the hydrogen concentration 35 percent and the concentration of the other reactant 10 percent by weight. The "widths" were assumed the same as in Table V. Given are the necessary temperatures for an energy production of 20 ergs/g sec. It is seen that all nuclei up to boron require extremely low temperatures in order not to give too much energy production; these temperatures (<10^7 degrees) are quite irreconcilable with the equations of hydrostatic and radiation equilibrium. On the other hand, oxygen and neon would require much too high temperatures. Only carbon and nitrogen require nearly, and nitrogen in fact exactly, the central temperature obtained from the Eddington integrations ($19 \cdot 10^6$ degrees). Thus from stellar data alone we could have predicted that the capture of protons by N^{14} is the process responsible for the energy production.

§8. REACTIONS WITH HEAVIER NUCLEI

Mainly for the sake of completeness, we shall discuss briefly the reactions of nuclei heavier than nitrogen. For the energy production, these reactions are obviously of no importance because the higher potential barrier of the heavier nuclei makes their reactions much less probable than those of the carbon-nitrogen group.

The most important point for a qualitative discussion is the question whether a $p - \alpha$ reaction is energetically possible for a particular nucleus, and, if possible, whether it is impeded by the potential barrier. In Table VII are listed the energy evolution in $p - \alpha$ reactions for all stable

(nonradioactive) nuclei up to chlorine. In the first column, the reacting nucleus is given, in the second, the product of a $p-\alpha$ reaction. The third column contains the reaction energy Q; when Q is negative, the reaction is energetically impossible so that the initial nucleus can only capture protons with γ-emission. In the fourth column, the height of the nuclear potential barrier is given for all reactions with positive Q. In the last column, the penetrability of the potential barrier is calculated according to standard methods (reference 10, p. 166). If $Q>B$, the penetrability is 1; if Q is negative, $P=0$ was inserted.

The *a priori* probability of a $p-\alpha$ reaction is roughly 10^4 times that of radiative capture. Therefore the emission of α-particles will be preferred when $P>10^{-4}$. It is seen from the table that for all nuclei up to boron the $p-\alpha$ reaction is strongly preferred, a fact which we recognized as the main reason for the impossibility of building up heavier elements than He^4 (§5). Furthermore, in the carbon group, only proton capture is possible for $C^{12}C^{13}N^{14}$ while for N^{15} the $p-\alpha$ reaction will strongly predominate (cf. §7).

The oxygen-fluorine group shows intermediate behavior. O^{16} can only capture protons, for O^{17} the capture and the α-emission will have roughly equal probability while for O^{18} and F^{19} the $p-\alpha$ reaction will be much more probable. With a ratio 10^4 for the *a priori* probabilities, about 40 percent of the O^{17} will become F^{18} (and then O^{18} by positron emission) while 60 percent will revert to N^{14}. Of the O^{18}, only 1 part in 2000 will become F^{19}, and of the F^{19}, only 1 in 10,000 will transform into Ne^{20}. Thus, under continued proton bombardment, about one O^{16} nucleus in $5 \cdot 10^7$ will ultimately transform into Ne^{20}, the rest will become nitrogen.

Actually, these considerations are somewhat academic because in general the supply of protons will be exhausted long before all the O^{16} initially present in the star will have captured a proton. Because of the small energy evolution in the reaction $O^{16}+H=F^{17}$, this reaction is extremely slow ($\sim 10^{12}$ years) so that equilibrium in the oxygen group will not be reached in astronomical times.

Among the nuclei heavier than fluorine, the $p-\alpha$ reaction is in general energetically permitted only for those with mass number $4n+3$. But even for these, the energy evolution is so much less than the height of the barrier that the penetrabilities are extremely small. Thus for all these elements only proton capture will occur (with the possible exception of Cl^{37}).

These considerations demonstrate the uniqueness of the carbon-nitrogen cycle.

§9. Agreement with Observations

In Table VIII, we have made a comparison of our theory (carbon-nitrogen reaction) with observational data for five stars for which such data are given by Strömgren.[1] The first five columns are taken from his table, the last contains the necessary central temperature to give the correct energy evolution with the carbon-nitrogen reactions (cf. Table VI). As in §7, we have assumed a N^{14} content of 10 percent, and an energy production at the center of ten times the average energy production (listed in the second column).

The result is highly satisfactory: The temperatures necessary to give the correct energy evolution (last column) agree very closely with the temperatures obtained from the Eddington integration (second last column). The only exception from this agreement is the giant Capella: This is not surprising because this star has greater luminosity than the sun at smaller density and temperature; such a behavior cannot possibly be explained by the same mechanism which ac-

TABLE VIII. *Comparison of the carbon-nitrogen reaction with observations.*

Star	Luminosity erg/g sec.	Central Density	H content (per- cent)	Central Temperature (Million degrees) Inte- gration	Energy Produc- tion
Sun	2.0	76	35	19	18.5
Sirius A	30	41	35	26	22
Capella	50	0.16	35	6	32
U Ophiuchi (bright)	180	12	50	25	26
Y Cygni (bright)	1200	6.5	80	32	30

[37] A. S. Eddington, *The Internal Constitution of the Stars* (Cambridge University Press, 1926).

counts for the main sequence. We shall come back to the problem of energy production in giants at the end of this section.

For the main sequence we observe that the small increase of central temperature from the sun to Y Cygni (19 to $33 \cdot 10^6$ degrees) is sufficient to explain the much greater energy production (10^4 times) of the latter. The reason for this is, of course, the *strong temperature dependence of our reactions* ($\sim T^{18}$, cf. §10). We may say that *astrophysical data themselves would demand such a strong dependence* even if we did not know that the source of energy are nuclear reactions. The small deviations in Table VIII can, of course, easily be attributed to fluctuations in the nitrogen content, opacity, etc.

In judging the agreement obtained, it should be noted that the "observational" data in Table VIII were obtained by integration of an Eddington model,[1, 37] i.e., the energy production was assumed to be almost constant throughout the star. Since our processes are strongly temperature dependent, the "point source" model should be a much better approximation. However, it seems that the results of the two models are not very different so that the Eddington model may suffice until accurate integrations with the point source model are available.[37a]

Since our theory gives a definite mechanism of energy production, it permits decisions on questions which have been left unanswered by astrophysicists for lack of such a mechanism. The first is the question of the "model," which is answered in favor of one approximating a point source model. The second is the problem of chemical composition. The equilibrium conditions permit for the sun a hydrogen content of either 35 or 99.5 percent when there is no helium, and intermediate values when there is helium. The central temperature varies from $19 \cdot 10^6$ to $9.5 \cdot 10^6$ when the hydrogen content increases

from 35 to 99.5 percent. It is obvious that the latter value can be definitely excluded on the basis of our theory: The energy production due to the carbon-nitrogen reaction would be reduced by a factor of about 10^8 (100 for nitrogen concentration, 10^6 for temperature). The proton combination (1) would still supply about 5 percent of the observed luminosity; but apart from the fact that a factor 20 is missing, the proton combination does not depend sufficiently on temperature to explain the larger energy production in brighter stars of the main sequence. Thus it seems that only a small range of hydrogen concentrations around 35 percent is permitted; what this range is, depends to some extent on the N^{14} concentration and also requires a more accurate determination of the distribution of temperature and density.

Next, we want to point out a rather well-known difficulty about the energy production of very heavy stars such as Y Cygni. With an energy production of 1200 ergs/g sec., and an available energy of $1.0 \cdot 10^{-5}$ erg per proton (formation of α-particles!), *all* the energy will be consumed in $1.7 \cdot 10^8$ years. Since at present Y Cygni still has a hydrogen content of 80 percent, its past life should be less than $3.5 \cdot 10^7$ years. We must therefore conclude that Y Cygni and other heavy stars were "born" comparatively recently—by what process, we cannot say. This difficulty, however, is not peculiar to our theory of stellar energy production but is inherent in the well-founded assumption that nuclear reactions are responsible for the energy production.[38]

Finally, we want to come back to the problem of stars outside the main sequence. The white dwarfs presumably offer no great difficulty. The internal temperature of these stars is probably rather low,[1, 39] because of the low opacity (degeneracy!) so that the small energy production may be understandable. Quantitative calculations are, of course, necessary. For the giants, on the other hand, it seems to be rather difficult to account for the large energy production by nuclear reactions. If the Eddington (or the point

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[37a] Mr. Marshak has kindly calculated the central temperature and density of the sun for the point source model, using Strömgren's tables for which we are indebted to Professor Strömgren. With an average atomic weight $\mu = 1$, Marshak finds

for the point source model $T_c = 20.3 \cdot 10^6$, $\rho_c = 50.2$
for the Eddington model $T_c = 19.6 \cdot 10^6$, $\rho_c = 72.2$

Not only is the temperature difference very small ($3\frac{1}{2}$ percent) but it is, for the sake of the energy production, almost compensated by a density difference in the opposite direction. The product $\rho_c T_c^{18}$ is only 20 percent greater for the point source model.

[38] Even if the most stable nuclei (Fe, etc.) are formed rather than He, the possible life will only increase by 30 percent.

[39] S. Chandrasekhar, Monthly Not. **95**, 207, 226, 676 (1935).

TABLE IX. *Relations between stellar constants.*

QUANTITY	$\gamma=18$ $\alpha=\frac{1}{2}$ $\beta=2\frac{3}{4}$ $\delta=19\frac{3}{4}$	$\gamma=18$ $\alpha=1$ $\beta=3\frac{1}{2}$ $\delta=20\frac{1}{2}$	$\gamma=15$ $\alpha=\frac{1}{2}$ $\beta=2\frac{3}{4}$ $\delta=16\frac{3}{4}$	$\gamma=4.5$ $\alpha=\frac{1}{2}$ $\beta=2\frac{3}{4}$ $\delta=6\frac{1}{4}$
Radius R	$M^{0.75}\mu^{0.57}(yz)^{0.05}$	$M^{0.71}\mu^{0.51}(yz)^{0.05}$	$M^{0.70}\mu^{0.49}(yz)^{0.06}$	$M^{0.20}\mu^{-0.36}(yz)^{0.16}$
Temperature T	$M^{0.25}\mu^{0.43}(yz)^{-0.05}$	$M^{0.29}\mu^{0.49}(yz)^{-0.05}$	$M^{0.30}\mu^{0.51}(yz)^{-0.06}$	$M^{0.80}\mu^{1.36}(yz)^{-0.16}$
Density ρ	$M^{-1.25}\mu^{-1.71}(yz)^{-0.15}$	$M^{-1.13}\mu^{-1.53}(yz)^{-0.15}$	$M^{-1.10}\mu^{-1.47}(yz)^{-0.18}$	$M^{0.40}\mu^{1.08}(yz)^{-0.48}$
Luminosity L	$M^{4.32}\mu^{6.04}y^{-1.06}z^{-0.06}$	$M^{5.15}\mu^{7.24}y^{-1.02}z^{-0.02}$	$M^{4.37}\mu^{6.13}y^{-1.07}z^{-0.07}$	$M^{5.00}\mu^{7.20}y^{-1.20}z^{-0.20}$
Surf. temp. T_S	$M^{0.70}\mu^{1.23}y^{-0.29}z^{-0.04}$	$M^{0.93}\mu^{1.55}y^{-0.28}z^{-0.03}$	$M^{0.74}\mu^{1.29}y^{-0.30}z^{-0.05}$	$M^{1.25}\mu^{1.98}y^{-0.38}z^{-0.13}$

source) model is used, the central temperatures and densities are exceedingly low, e.g., for Capella $T=6\cdot10^6$, $\rho=0.16$. Only a nuclear reaction going at very low temperature is therefore at all possible; $Li^7+H=2He^4$ would be just sufficient. But it seems difficult to conceive how the Li^7 should have originated in *all* the giants in the first place, and why it was not burned up long ago. The only other source of energy known is gravitation, which would require a core model[40] for giants.[41] However, any core model seems to give small rather than large stellar radii.

§10. THE MASS-LUMINOSITY RELATION

In this section, we shall use our theory of energy production to derive the relation between mass and luminosity of a star. For this purpose, we shall employ the well-known homology relations (reference 1, p. 492). This is justified because we assume that all stars have the same mechanism of energy evolution and therefore follow the same model. Further, it is assumed that the matter throughout the star is non-degenerate which seems to be true for all stars except the white dwarfs. (For all considerations in this and the following two sections, cf. reference 1.)

We shall consider the mass of the star M, the mean molecular weight μ, the concentration of "Russell Mixture" y and the product of the concentrations of hydrogen and nitrogen, z, as independent variables. In addition, we introduce for the moment the radius R which, however, will be eliminated later. Then, obviously, we have for the density (at each point)

$$\rho \sim M/R^3. \qquad (46)$$

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[40] L. Landau, Nature **141**, 333 (1938).
[41] This suggestion was made by Gamow in a letter to the author.

From the equation of hydrostatic equilibrium

$$dp/dr = -GM_r\rho/r^2 \qquad (47)$$

($G=$ gravitational constant) and the gas equation

$$p=RT\rho/\mu \qquad (47a)$$

(R the gas constant, radiation pressure neglected), we find

$$T\sim M\mu/R. \qquad (48)$$

Finally, we must use the equation of radiative equilibrium:

$$aT^3\frac{dT}{dr} = -\frac{3}{4}\frac{k}{c}\rho\frac{L_r}{4\pi r^2}, \qquad (49)$$

where a is the Stefan-Boltzmann constant, c the velocity of light, k the opacity, and L_r the luminosity (energy flux) at distance r from the center.

For the opacity, we assume

$$k\sim y\rho^\alpha T^{-\beta} \qquad (50)$$

(y concentration of heavy elements). Usually, α is taken as 1 and $\beta=3.5$ (Kramers' formula). However, the Kramers formula must be divided by the "guillotine factor" τ which was calculated from quantum mechanics by Strömgren.[42] For densities between 10 and 100, and temperatures between 10^7 and $3\cdot10^7$, Strömgren's numerical results can be fairly well represented by taking $\tau\sim\rho^{\frac{1}{3}}T^{-\frac{1}{2}}$. Therefore we adopt $\alpha=\frac{1}{2}$, $\beta=2.75$ in (50).

The luminosity may be written

$$L\sim M\rho zT^\gamma. \qquad (51)$$

That the energy production per unit mass, L/M, contains a factor ρ follows from the fact that it is due to two-body nuclear reactions; this factor is
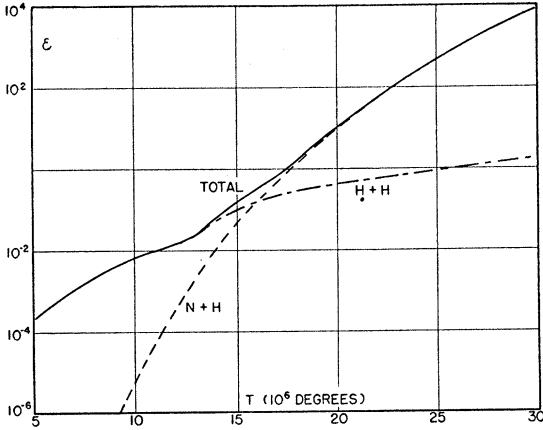
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[42] Cf. reference 1, Table 6, p. 485.

FIG. 1. The energy production in ergs/g sec. due to the proton-proton combination (curve H+H) and the carbon-nitrogen cycle (N+H), as a function of the central temperature of the star. Solid curve: total energy production caused by both reactions. The following assumptions were made: central density = 100, hydrogen concentration 35 percent, nitrogen 10 percent; average energy production 1/5 of central production for H+H, 1/10 for N+H.

apparent from all our formulae, e.g., (4). $z = x_1 x_2$ is the product of the concentrations of the reacting nuclei (N^{14} and H). For γ we obtain from (4), (6)

$$\gamma = \frac{d \log (\tau^2 e^{-\tau})}{d \log T} = \tfrac{1}{3}(\tau - 2). \qquad (52)$$

For $N^{14}+H$ and $T = 2 \cdot 10^7$, this gives $\gamma = 18$. For $T = 3.2 \cdot 10^7$ (Y Cygni), $\gamma = 15.5$; generally, $\gamma \sim T^{-\frac{1}{3}}$. $\gamma = 18$ will be a good approximation over most of the main sequence.

Inserting (50), (51) in (49), we have

$$T^{4+\beta-\gamma} \sim yzM\rho^{2+\alpha}R^{-1}. \qquad (53)$$

Combining this with (46), (48), and introducing the abbreviation

$$\delta = \gamma + 3 + 3\alpha - \beta \qquad (54)$$

we find

$$R \sim M^{1-2(2+\alpha)/\delta}\mu^{1-(7+3\alpha)/\delta}(yz)^{1/\delta}, \qquad (55)$$

$$T \sim M^{2(2+\alpha)/\delta}\mu^{(7+3\alpha)/\delta}(yz)^{-1/\delta}, \qquad (56)$$

$$\rho \sim M^{-2+6(2+\alpha)/\delta}\mu^{-3+3(7+3\alpha)/\delta}(yz)^{-3/\delta}, \qquad (57)$$

$$L \sim M^{3+2\alpha+2(2+\alpha)(\beta-3\alpha)/\delta}\mu^{4+3\alpha+(7+3\alpha)(\beta-3\alpha)/\delta}$$

$$\times y^{-1}(yz)^{-(\beta-3\alpha)/\delta}. \qquad (58)$$

Furthermore, the surface temperature may be of

interest, we have

$$T_s \sim L^{\frac{1}{2}}R^{-\frac{1}{2}} \sim M^{\frac{1}{2}+\frac{1}{2}\alpha+\frac{1}{2}(2+\alpha)(\beta-3\alpha+2)/\delta}$$

$$\times \mu^{\frac{1}{2}+\frac{3}{4}\alpha+\frac{1}{4}(7+3\alpha)(\beta-3\alpha+2)/\delta}y^{-\frac{1}{4}}$$

$$\times (yz)^{-\frac{1}{4}(\beta-3\alpha+2)/\delta}. \qquad (59)$$

In Table IX, these formulas are given explicitly, for four different sets of constants.

The most important result is that the central *temperature* depends only slightly on the mass of the star, viz. as $M^{0.25}$ and $M^{0.30}$ for $\gamma = 18$ and 15. The reason for this is the strong temperature dependence of the reaction rate: The exponent of M in (56) is inversely proportional to δ which is mainly determined (cf. (54)) by the exponent γ in formula (51) for the temperature dependence of the reaction rate. The integration of the Eddington equations with the use of observed luminosities, radii, etc., gives, in fact, only a small dependence of the central temperature on the mass. This can *only* be explained by a strong temperature dependence of the source of stellar energy, a fact which has not been sufficiently realized in the past. Theoretically, the central temperature increases somewhat with increasing mass of the star, more strongly with the mean molecular weight, and is practically independent of the chemical composition, i.e., of y and z.

The *radius* of the star is larger for heavy stars and for high molecular weight. The density behaves, of course, in the opposite way. Both these results are in qualitative agreement with observation. The product of mass and density which occurs in Eq. (51) for the luminosity, is almost independent of the mass; therefore, for constant concentrations z, the luminosity is determined by the central temperature alone. Both radius and density are almost independent of the chemical composition, except insofar as it affects μ.

The *luminosity* increases slightly faster than the fourth power of the mass and the sixth power of the mean molecular weight. This increase is considerably less than that usually given ($M^{5.5}\mu^{7.5}$) and agrees better with observation. The difference from the usual formula is mainly due to the different dependence of the opacity on density and temperature; in fact, with the usual assumption ($\alpha = 1$, $\beta = 3\frac{1}{2}$), we obtain $M^{5.15}\mu^{7.24}$.

The remaining difference is that the dependence on the radius is carried as a separate factor in the usual formula while we have expressed R in terms of M and μ. The observations show, for bright stars, an even slower increase than M^4; this seems to be due to the lower average molecular weight (higher hydrogen content) of most very bright stars. It may be that these stars become unstable because of excessive energy production when their hydrogen content becomes too low (cf. §12).—The luminosity is inversely proportional to the concentration y of heavy elements (Russell mixture) because y determines the opacity. However, L is almost independent of the nitrogen concentration, as are R, T and ρ.

All these considerations are valid for bright stars, down to about three magnitudes fainter than the sun. For fainter stars, with lower central temperatures, the proton combination $H+H = D + \epsilon^+$ should become more probable than the carbon-nitrogen reactions, because this reaction depends less on temperature. In discussing the energy production from $H+H$, we must take into account that (cf. §5) at low temperatures, this reaction leads only to He^3 rather than He^4, because of the slowness of the reaction $He^4 + He^3 = Be^7$. (Assumption B of §5 is made, viz. that He^3 is more stable than H^3, and Li^4 unstable.) This causes a rather sudden drop in the energy evolution from $H+H$ around $14 \cdot 10^6$ degrees (cf. Fig. 1), i.e., just below the temperature at which the proton combination becomes important ($\sim 16 \cdot 10^6$ degrees, see Fig. 1). Therefore, the temperature exponent γ stays fairly large (~ 13, cf. Fig. 2) down to about $13 \cdot 10^6$ degrees which corresponds to an energy production of about one percent of that of the sun (five magnitudes fainter). For still fainter stars, γ drops to very low values, reaching a minimum of about 4.5 near 10^7 degrees.

The relations between central temperature, radius, luminosity and mass for this case ($\gamma = 4.5$) are given in the last column of Table IX. The temperature is seen to depend much more strongly on the mass (as $M^{0.80}$) while the radius becomes almost independent of M and the density decreases with decreasing mass. The luminosity decreases faster with the mass (as M^5) than for the bright stars. Unfortunately, little material is available for these fainter stars. This

is the more regrettable as it is rather important for nuclear physics to decide whether the $H+H$ reaction is really as probable as assumed in this paper: There is some possibility that it is forbidden by selection rules (cf. reference 16, p. 250) in which case it would be about 10^6 times *less* probable. Then the carbon-nitrogen reactions would furnish the energy even for faint stars, and the central temperature of these stars would not depend much on their mass.

Figure 1 gives the energy production due to the proton combination ($H+H$) and to the carbon-nitrogen reactions ($N+H$) as a function of the central temperature, on a logarithmic scale. The great preponderance of $H+H$ at low and $N+H$ at high temperatures is evident. The following assumptions were made: Hydrogen concentration 35 percent, nitrogen 10 percent, central density $\rho = 100$, average energy production $= \frac{1}{5}$ of central production for $H+H$, $\frac{1}{10}$ for $N+H$ reaction. He^3 is supposed to be more stable than H^3, and Li^4 unstable (assumption B of §5).
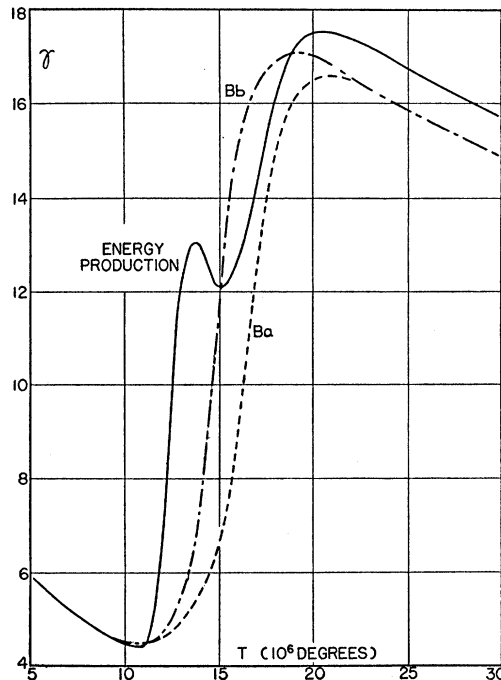

Fig. 2. The exponent γ in the relation $L \sim T^\gamma$ between luminosity L and temperature T, as a function of T. Solid curve: γ for total energy production (logarithmic derivative of solid curve in Fig. 1). Dotted curves: γ for stability against temperature changes (curve Ba for times less than 14 months, Bb for more than 14 months).

Figure 2, solid curve, gives the exponent γ in the *total* energy production. It is low at low temperatures ($\sim$4–5, hydrogen reaction) and has a minimum of 4.44 at 11 million degrees. Between 11 and 14 million degrees, γ rises steeply as the reaction $He^4 + He^3 = Be^7$ sets in (§5); then it falls again from 13 to 12 when this reaction reaches saturation. From $T = 15$, the carbon-nitrogen reactions set in, causing a rise of γ to a maximum of 17.5 at $T = 20$, while at higher temperatures γ decreases again as $T^{-\frac{1}{2}}$.

§11. Stability Against Temperature Changes

Cowling[43] has investigated the stability of stars against vibration. This stability is determined mostly by the ratio γ of the specific heats at constant pressure and constant volume. If the radiation pressure in the star is negligible compared with the gas pressure ($\gamma = 5/3$) then the star will be stable for any value of the temperature exponent n in the energy production (38), up to $n \approx 450$. Only for very heavy stars, for which the radiation pressure is comparable with the gas pressure, does the stability condition put a serious restriction on the temperature dependence of the energy production. According to our theory, the energy production is proportional to T^{17} (see below); according to Cowling, stability will then occur when $\gamma > 10/7$. The corresponding ratio of radiation pressure to total pressure is

$$1 - \beta = \frac{5 - 3\gamma}{3(7\gamma - 9)} = 0.24 \qquad (60)$$

and the corresponding mass of the star, according to Eddington's "standard model," is $17/\mu^2 \odot$, where μ is the average molecular weight. Therefore practically all the stars for which good observational data are available will be stable.

It may be worth while to point out that the temperature exponent n for these stability considerations is not exactly equal to that for the energy production in equilibrium. A change of temperature gives rise to radial vibrations of the star whose period is of the order of days or, at most, a few years. On the other hand, when the temperature is raised or lowered, the equilibrium

[43] T. G. Cowling, Monthly Not. **94**, 768 (1934); **96**, 42 (1935).

between carbon and nitrogen will be disturbed and it takes a time of the order of the lifetime of C^{12} ($\sim 10^6$ years) to restore equilibrium at the new temperature. Thus we must take the concentrations of carbon and nitrogen corresponding to the *original* temperature.

At $T = 2 \cdot 10^7$ degrees, the carbon reactions have a $\tau = 50.6$ (Table V) and therefore a temperature exponent $\gamma_C = 16.2$ (cf. 52); the nitrogen reactions have $\tau = 56.3$ and $\gamma_N = 18.1$. The number of reactions per second is, in equilibrium, the same for each of the reactions in the chain (42). The energy evolution in the first three reactions together is 11.7 mMU, after subtracting 0.8_5 mMU for the neutrino emitted by N^{13}. These three reactions are carbon reactions, the remaining three, with an energy evolution of 15.0 mMU, are nitrogen reactions. Thus the effective temperature exponent for stability problems becomes

$$\gamma = \frac{11.7\gamma_C + 15.0\gamma_N}{26.7} = 17.3 \qquad (60a)$$

at $2 \cdot 10^7$ degrees. γ is approximately proportional to $T^{-\frac{1}{2}}$ (cf. 52, 6).

For the proton combination, we have to distinguish the three possibilities discussed in §5. The simplest of these is assumption C.

Assumption C: Li⁴ unstable, H³ more stable than He³

In this case, there are two "slow" processes, *viz.* (a) the original reaction $H + H$ and (b) the transformation of He^3 into H^3 by electron capture ($\sim$2000 years). (a) depends on temperature approximately as $T^{3.5}$, (b) is independent of T. The energy evolution up to the formation of He^3 is 7.2 mMU, from He^3 to He^4 21.3 mMU. Thus

$$\gamma = \frac{7.2 \cdot 3.5 + 21.3 \cdot 0}{28.5} = 0.9. \qquad (61C)$$

This would be a very slight dependence indeed.

Assumption A: Li⁴ stable

According to Table V, the transformation of He^3 into Li^4 takes about one day.

(a) For times *shorter than one day*, the reactions up to He^3 and from then on are independent of each other. The first group again has $\gamma_H = 3.5$ and gives 7.2 mMU, the second group now gives only 8.3 mMU because the remaining 13.0 are lost to the neutrino from Li^4 (cf. §5) and has (Table V) $\gamma_{He} = 6.9$. Therefore

$$\gamma = \frac{7.2 \cdot 3.5 + 8.3 \cdot 6.9}{15.5} = 5.3. \qquad (61Aa)$$

(b) For times *longer than one day*, the proton combination determines the whole chain of reactions so that

$$\gamma = 3.5. \qquad (61\text{Ab})$$

Assumption B. Li^4 unstable, He^3 more stable than H^3 (most probable assumption)

As was pointed out in §5, the reaction $He^4 + He^3 = Be^7$ is so slow ($3 \cdot 10^7$ years, Table V) that the concentration of He^3 will remain unaffected by the temperature fluctuation. Be^7 has a mean life of 14 months at the center of the sun so that there are again two cases:

(a) *For times less than 14 months*, there are three groups of reactions, (a) those up to He^3, giving again 7.2 mMU with $\gamma_H = 3.5$, (b) the reaction $He^4 + He^3 = Be^7$, giving (Table V) 1.6 mMU with $\gamma_{He} = 15.1$, and (c) the electron capture by Be^7, followed by $Li^7 + H = 2He^4$. This last reaction does not depend on temperature ($\gamma_{Be} = 0$) and gives 18.6 mMU. Therefore the effective γ becomes

$$\gamma = \frac{7.2 \cdot 3.5 + 1.6 \cdot 15.1 + 18.6 \cdot 0}{27.4} = 1.8. \qquad (61\text{Ba})$$

(b) *For times longer than 14 months*, the reaction $He^4 + He^3 = Be^7$ governs all the energy evolution from He^3 on, so that

$$\gamma = \frac{7.2 \cdot 3.5 + 20.2 \cdot 15.1}{27.4} = 12.1 \qquad (61\text{Bb})$$

at $2 \cdot 10^7$ degrees.

At low temperatures, the reaction $He^4 + He^3 = Be^7$ stops altogether (cf. §5) so that then the γ of the $H + H$ reaction itself determines the radial stability. From 12 to $16 \cdot 10^6$ degrees we have a transition region in which the importance of the $He^4 + He^3$ reaction (and the consequent ones) is reduced.

In Fig. 2, curves Ba and Bb, we have plotted the effective γ for the radial stability, by taking into account both $C + N$ and $H + H$ reactions, and making the same assumptions about the concentrations of hydrogen and nitrogen as in Fig. 1 (cf. end of §10). Assumption B was made regarding the stability of Li^4 and He^3; the curves Ba and Bb refer to the short time and long time formulas (61Ba) and (61Bb). At high temperatures, the two curves coincide because

then the proton combination is unimportant compared with the carbon-nitrogen reactions. The combined γ is seen to reach a maximum of 17 (for the long time curve).

§12. STELLAR EVOLUTION[44]

We have shown that the concentrations of heavy nuclei (Russell mixture) and, therefore, also of nitrogen, cannot change appreciably during the life of a star. The only process that occurs is the transformation of hydrogen into helium, regardless of the detailed mechanism. The state of a star is thus described by the hydrogen concentration x, and by a *fixed* parameter, y, giving the concentration of Russell mixture. The rest, $1 - x - y$, is the helium concentration. Without loss of generality, we may fix the zero of time so that the helium concentration is zero. (Then the actual "birth" of the star may occur at $t > 0$).

It has been shown (Table IX) that the luminosity depends on the chemical composition practically only[45] through the mean molecular weight μ. This quantity is given by

$$1/\mu = 2x + \tfrac{3}{4}(1 - x - y) + \tfrac{1}{2}y = (5/4)(x + a), \qquad (62)$$

$$a = 0.6 - 0.2y, \qquad (62\text{a})$$

taking for the molecular weights of hydrogen, helium and Russell mixture the values $\tfrac{1}{2}$, $4/3$ and 2, respectively.[1]

Now the rate of decrease of the hydrogen concentration is proportional to the luminosity, which we put proportional to μ^n. According to Table IX, n is about 6. Then

$$dx/dt \sim -(x + a)^{-n}. \qquad (63)$$

Integration gives

$$(x + a)^{n+1} = A(t_0 - t), \qquad (64)$$

where A is a constant depending on the mass and other characteristics of the star. Since $x = 1 - y$ at $t = 0$, we have

$$At_0 = (1.6 - 1.2y)^{n+1}. \qquad (64\text{a})$$

[44] Most of these considerations have already been given by G. Gamow, Phys. Rev. **54**, 480(L) (1938).

[45] Except for the factor y^{-1} which, however, does not change with time.

It is obvious from (63) and (64) that the hydrogen concentration decreases slowly at first, then more and more rapidly. E.g., when the concentration of heavy elements is $y=\frac{1}{2}$, the first half of the hydrogen in the star will be consumed in 87 percent of its life, the second half in the remaining 13 percent. If the concentration of Russell mixture is small, the result will be even more extreme: For $y=0$, it takes 92 percent of the life of the star to burn up the first half of the hydrogen. Consequently, very few stars will actually be found near the end of their lives even if the age of the stars is comparable with their total lifespan t_0 (cf. 64a). In reality, the lifespan of all stars, except the most brilliant ones, is long compared with the age of the universe as deduced from the red-shift ($\sim 2 \cdot 10^9$ years): E.g., for the sun, only one percent of the total mass transforms from hydrogen into helium every 10^9 years so that there would be only 2 percent He in the sun now, provided there was none "in the beginning." The prospective future life of the sun should according to this be $12 \cdot 10^9$ years.

It seems to us that this comparative youth of the stars is one important reason for the existence of a *mass*-luminosity relation—if the chemical composition, and especially the hydrogen content, could vary absolutely at random we should find a greater variability of the luminosity for a *given* mass.

It is very interesting to ask what will happen to a star when its hydrogen is almost exhausted. Then, obviously, the energy production can no longer keep pace with the requirements of equilibrium so that the star will begin to contract.

(This is, in fact, indicated by the factor $z^{1/8}$ in Eq. (55) for the stellar radius; z is proportional to the hydrogen concentration.) Gravitational attraction will then supply a large part of the energy. The contraction will continue until a new equilibrium is reached. For "light" stars of mass less than $6\mu^{-2}$ sun masses (reference 1, p. 507), the electron gas in the star will become degenerate and a white dwarf will result. In the white dwarf state, the necessary energy production is extremely small so that such a star will have an almost unlimited life. This evolution was already suggested by Strömgren.[1]

For heavy stars, it seems that the contraction can only stop when a neutron core is formed. The difficulties encountered with such a core[46] may not be insuperable in our case because most of the hydrogen has already been transformed into heavier and more stable elements so that the energy evolution at the surface of the core will be by gravitation rather than by nuclear reactions. However, these questions obviously require much further investigation.

These investigations originated at the Fourth Washington Conference on Theoretical Physics, held in March, 1938 by the George Washington University and the Department of Terrestrial Magnetism. The author is indebted to Professors Strömgren and Chandrasekhar for information on the astrophysical data and literature, to Professors Teller and Gamow for discussions, and to Professor Konopinski for a critical revision of the manuscript.

[46] G. Gamow and E. Teller, Phys. Rev. **53**, 929(A), 608(L) (1938).